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В сборнике опубликованы статьи участников конференции, посвященные исследованиям в области математического программирования, дискретной оптимизации, теории игр, оптимизации в машинном обучении и анализе данных, приложений в исследовании операций: задачи составления расписаний, маршрутизации, размещения предприятий, упаковки, управления производством и компьютерными системами.

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Содержание

Надежда Дресвянская[1]

Локальный поиск в задаче выпуклого квадратичного двухуровневого программирования

[1] Melentiev Energy Systems Institute SB RAS

В работе рассматривается задача выпуклого квадратичного двухуровневого программирования, возникающая в результате моделирования поведения генерирующих компаний на рынке электроэнергии. Для решения поставленной задачи предлагается метод локального поиска, основанный на применении квадратичных опорных функций. Метод основан на решении последовательности задач квадратичной оптимизации и использует специфические свойства исследуемой задачи. Обосновывается сходимость метода к точке локального минимума. Приводятся результаты вычислительных экспериментов.

Keywords: двухуровневая оптимизация, локальный поиск, опорные функции.

Виктория Мунько[1], **Анна Зыкина**

Цифровое решение для моделирования и оптимизации программы обучения

[1] Омский государственный технический университет

В действующих системах управления учебным процессом отсутствует интеллектуальный сервис, который может помочь образовательным учреждениям как разрабатывать актуальные в данный момент востребованные образовательные программы, так и адаптировать их к стремительно меняющимся требованиям к профессиональной сфере и профессорско-преподавательскому составу (ППС). В работе предлагается программное решение для моделирования и оптимизации формирования учебного плана образовательного направления с учетом профессиональных компетенций и состава ППС. Формирование учебного плана представляет собой сложную слабо формализованную задачу в системе управления учебным процессом. Эффективное функционирование такой системы зависит от планирования информационных и организационных ресурсов, используемых методов и средств для оптимизации процессов системы. Все это приводит к многообразию вариантов формирования учебного плана, причем выбор конкретного решения зависит от субъективных факторов и при попытке их формализации – от принятых моделей и алгоритмов. Существующие разработки по тематике исследования сводятся лишь к формализации связи дисциплин и компетенций, при этом отсутствует связь дисциплин и преподавателей. На основе выделенных факторов, влияющих на формирование учебного плана образовательной программы, предложена математическая модель формирования учебно-

го плана в виде двухэтапной схемы с учетом выбора дисциплин, профессиональных компетенций и состава ППС. Задача первого этапа задает взаимосвязь дисциплин и компетенций. Ограничения задачи первого этапа связаны с нормативными требованиями образовательных стандартов по обеспечению компетенций и по количеству часов в учебном плане. Критерием первого этапа является максимизация обеспеченности каждой заданной компетенции. Задача второго этапа задает взаимосвязь дисциплин и преподавателей. Ограничения задачи второго этапа также связаны с нормативными требованиями образовательных стандартов: это требования к квалификации ППС и к количеству часов у преподавателя в учебном плане. Критериями второго этапа являются максимизация профильности преподавателей и минимизация затрат на реализацию учебного процесса. Реализация выбора дисциплин и обеспечивающих их преподавателей по представленной модели поможет разработке востребованных образовательных программ обучения при снижении трудозатрат специалистов учебно-методического управления. Предложенная концепция формирования программ обучения может быть использована для организации программ дополнительного профессионального образования на предприятиях.

Работа выполнена в рамках Государственного задания № FSGF-2024-0006 «Разработка новых моделей и методов принятия решений в многоуровневых системах управления».

Keywords: учебный план, математическое моделирование.

Иван Шарун[1], **Анна Зыкина**

Подход к обучению нейронных сетей с помощью вариационных неравенств

[1] Омский государственный технический университет

В глубоком обучении зачастую целевая функция является невыпуклой. Ключевой особенностью данного доклада является использование аппарата вариационных неравенств для решения задачи обучения нейронных сетей. Разработчики получают инструмент со знакомым интерфейсом, но классическая оптимизационная задача заменяется инструментарием вариационных неравенств, способным решать задачи невыпуклой оптимизации эффективно.

Представлен подход, который отказывается от распространенной схемы вычисления оптимальных параметров нейронной сети с использованием метода градиентного спуска. Задача обучения одного слоя нейронной сети ставится как задача решения вариационного неравенства, которая гарантирует сходимость к оптимальному решению.

Полученные в результате работы алгоритмы могут использоваться как

для обучения всей сети, так и для более эффективной точной настройки предварительно обученной нейронной сети, что является важным шагом для развертывания многих моделей машинного обучения, таких как большие языковые модели.

Алгоритм представляет из себя процесс итеративного обучения, где на каждой итерации берется выборка из множества объектов обучения, после производится применение линейного оператора, на выходе которого мы получаем результаты последнего выходного слоя нейронной сети, далее производится расчет ошибки в пространстве выходного слоя, после происходит применение сопряженного оператора, который преобразовывает результаты пространства выходного слоя в результаты пространства весов и так для каждого элемента выборки. Элементы, которые не попали в выборку, перенимают значения с предыдущих итераций. После данных операций, в рамках каждой итерации, производится обновление весов с помощью проекции матриц обновленных весов на матрицу предыдущих весов.

В докладе презентуется разработанная библиотека, основанная на аппарате вариационных неравенств, библиотека интегрирована с инструментарием PyTorch для создания и использования нейронных сетей. Работа выполнена в рамках Государственного задания № FSGF-2024-0006 «Разработка новых моделей и методов принятия решений в многоуровневых системах управления».

Keywords: вариационные неравенства, глубокое обучение, послойное обучение.

Mostafa Abotaleb, Tatiana Makarovskikh[1]

Solving the optimizing parameters problem for non-linear datasets using the High-Order General Least Deviations Method (GLDM) algorithm

[1] South Ural State University

This study rigorously explores the Generalized Least Deviation Method (GLDM) for forecasting across various datasets, focusing on its adeptness in determining model coefficients a_i for orders two to five. Our analysis highlights the method's flexibility and precision, demonstrating that the second-order model excels with smaller datasets through statistical significance, while the fifth-order model, with its minimal error metrics and heightened explanatory prowess, is superior for larger datasets. This adaptability underscores GLDM's efficacy in forecasting, tailored to dataset specifics. The model's validity is assessed via detailed statistical methods, where ANOVA tests validate the overall model significance, and t-tests confirm the individual coefficients' a_i significance. These examinations definitively advocate for GLDM's utility in

refining forecasting models, presenting substantial implications for predictive analytics in diverse fields.

Keywords: Mathematical model, Least deviations, residuals, Forecasting, Time Series, quasilinear model.

Mirali Agaev, Dmitry Arkhipov

Amateur sports league scheduling

The sports business has largely developed over the last few years, involving new teams, players, supporters, and sponsors. Professional sports leagues in popular sports like football or hockey need to take into account a lot of factors while scheduling matches. Days to choose a match need to be selected with respect to the calendar of national and local holidays, international tournaments, team routing during the away match tour, and rest time between the consecutive matches. Match times should be chosen to increase the number of visitors and online translation viewers, car traffic, and public transport schedule. Such problems require large and flexible modeling. S. Knust used a graph-based approach to create a schedule for the Bundesliga, and Gurobi used IP modeling to solve this problem for the USA National Football League. In recent years, amateur sports leagues have also become more popular. For example, the Continental Football League (cfl.lfl.ru) last year involved more than 250 thousand players who took part in 80 thousand games. Most matches in such leagues are played in the same city on weekends or after work. So the set of constraints is very different from professional sports. Its business models are based on team contributions, so scheduling objectives must be focused on participant comfort, unlike viewer numbers in professional sports. In this research, we consider different statements of scheduling problem of amateur round-robin tournaments. Considered scenarios cover the assignments of teams and referees for matches, the selection of time, and the stadium to hold them. Most constraints and objectives are related to supporting schedule feasibility and improving participant experiences. For a simple statement, we demonstrate that a problem can be solved by an algorithm with polynomial complexity. For more complex scenarios, we consider several approaches: integer programming, constraint programming, and self-made graph-based heuristics. Numerical experiments on real-world scenarios were conducted to compare the efficiency of proposed approaches.

Keywords: scheduling, round-robin tournament, combinatorial optimization, graph edge coloring, metaheuristics.

Kamil Aida-zade[1], Samir Quliyev[2]

Automated and Automatic Systems of Management of an Optimization Programs Package for Decisions Making

[1] Institute of Control Systems of the Ministry of Science and Education of Republic of Azerbaijan

[2] Institute of Control Systems of the Ministry of Science and Education of the Azerbaijan Republic

It is known that, in spite of a large number of methods for numerical solutions to various classes of problems, the choice of the most efficient method for solving a particular problem under specific values of its parameters requires a large number of comparative experiments. As a rule, end users tend to have difficulty both in carrying out such experiments, which require knowledge of the domain of applicability of various numerical methods, and in properly conducting the comparative analysis of the results, which is quite time-consuming. We carry out the analysis of techniques and algorithms for managing a computational process to solve difficult or complex optimization problems with the use of computer systems. In this work, for the class of multivariate optimization problems, we propose two approaches for facilitating the use of available applied software packages using modern multi-processor and/or multicore computer systems. One of the approaches involves the active work of the user with the optimization program package in dialogue mode. The other approach involves packet control by means of a specially developed control program in automatic mode. The work contains the protocols and results of computer-based experiments for the class of un-constrained optimization problems on the basis of the developed software package.

Keywords: Decision Making, Optimization Methods, Optimal Control Methods, Parallel Computing, Multiprocessor/Multicore Systems, Dialog Systems.

Farid Akhmatshin[1], **Lev Kazakovtsev**

Mini-batch K-means++ clustering initialization

[1] Reshetnev Siberian State University of Science and Technology

Fast algorithms for clustering large and ultra-large volumes of data are in demand in various fields, such as, for example, vector databases. The k-means algorithm remains the most popular, however, it is extremely dependent on the initialization method. K-means++ is a well-established procedure for selecting initial cluster centers (centroids) for the k-means algorithm. In this work, we present a new algorithm capable of solving the k-means++ initialization problem for obtaining a good initial solution. The well-known k-means++ algorithm works well for data sets of N data vectors in a d-dimensional space when perform- ing a single pass through the input data in $O(k)$ iterations, and each iteration is characterized by complexity $O(Ndk)$, where k is the number

of centroids (clusters). So, the total execution time is $O(Ndk2)$. Since k-means++ requires k passes through the data to initialize, it does not scale well to large data sets. We propose the Mini-Batch K-means++ algorithm with a single pass through the data and a total expected execution time of $O(dk(b+\log B))$, where B is the number of batches of data, and b is the number of data vectors in one batch, $N \leq Bb$. The higher comparative efficiency of the new Mini-Batch k-means++ algorithm for big data is shown by experiment. The minimum amount of memory resources for the algorithm to run is expected at $B = n1/2$ with a non-optimal execution time of $O(dk(b + \log B))$. In the optimal scenario for running a new algorithm, for $2w$ data vectors, the expected time expenditure is $O(dkw)$. It is shown that the quality of solutions is not inferior to classical k-means++.

Supported by Ministry of Science and Higher Education of the Russian Federation (Project FEFE-2023-0004).

Keywords: clustering, mini-batch k-means, k-means++.

Mohammad Alkousa[1], **Fedor Stonyakin**[2], **Asmaa Abdo, Mohammad Alcheikh**

Optimal Convergence Rate for Mirror Descent Methods with Special Time-Varying Step Sizes Rules

[1] Moscow Institute of Physics and Technology

[2] Moscow Institute of Physics and Technology

In this paper, the optimal convergence rate without the presence of a logarithmic factor is proved for mirror descent methods with special time-varying step sizes for solving classical constrained non-smooth problems, and problems with the composite model. The proven result is an improvement on the well-known rate $O\left(N^{-1/2}\log(N)\right)$ (N is the total number of iterations performed by the algorithm) for the mirror descent algorithms with the time-varying step sizes under consideration. It was studied a new weighting scheme that assigns smaller weights to the initial points and larger weights to the most recent points. This scheme improves the convergence rate of the studied mirror descent methods which in the conducted numerical experiments outperform the other methods providing a better solution in all the considered test problems.

Keywords: Convex Optimization, Non-Smooth Problem, Mirror Descent, Mirror-C Descent, Time-varying step size, Optimal Convergence Rate, Sub-Optimal Convergence Rate.

Anatoly Antipin, Elena Khoroshilova

Controlled dynamics with a boundary-value problem on calcu-

LATING A FIXED POINT OF AN EXTREME MAPPING

An optimal control problem is considered on a fixed time interval. The left end of the phase trajectory is fixed, and the right end is given implicitly as a solution to the linear programming problem. The boundary value problem is formed as a problem of calculating a fixed point of an extremal mapping. The problem requires choosing a control to construct a phase trajectory in Hilbert space that leaves a given point (the initial condition at the left end) and ends up at a fixed point of the extremal mapping at the right end of the time interval. The paper proposes and develops an evidence-based method for solving the optimal control problem. The method is based on sufficient optimality conditions. They ensure the convergence of the computational process to solve the problem. The resulting solution is a demonstrably guaranteed result, in contrast to solutions obtained using the Maximum Principle, which is only a necessary condition for optimality. Within the framework of functional analysis, the existence of a fixed point of a given extremal transformation is proved. We also prove the convergence of the computational algorithm in all variables of the problem, namely strong convergence (in the norm of space) for phase and conjugate trajectories, strong convergence in the variables of the boundary value problem, and weak convergence in control. Only evidence-based computing technologies transform mathematical models into a tool for obtaining guaranteed solutions.

Keywords: optimal control, Lagrange function, boundary value problems, duality, saddle point methods, convergence.

Sergey Antsyz

On Refinement for Analysis of Two Forms of Taxation

The object of the study is a complex economic system consisting of the state and taxpayers. In [1], based on the Solow equation [2], it was shown that from the point of view of the state, the flat scale of property taxation unexpectedly exceeds progressive taxation. The progressive tax rate in this article was assumed to be linearly dependent on the size of the tax base. In further studies, based on the same progressive tax model, similar results were obtained from a comparative analysis of two forms of income tax and consumption tax. In [3], a discrete model of the economy is described, with the progressive tax rate is a piecewise constant function. Such modeling of taxation is more adequate to economic reality. In this work, it is planned to use the progressive tax model proposed in [3] to clarify the comparative analysis of two forms of taxation: flat scale and progressive tax. It is also planned to consider economies that include stratums of taxpayers with small and large incomes.

The author was supported by the Ministry of Science and Higher Educa-

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Keywords: growth economic models, flat scale, progressive tax, two stratums of taxpayers.

Konstantin Baikov, Bogdan Kornitskii, Andrey Melnikov[1]

Approximate algorithm for multi-criteria optimization models with implicitly defined functions

[1] Sobolev Institute of Mathematics

The paper presents the modeling library and the solver for multi-objective optimization problems with implicitly defined objective functions and constraints. It is assumed that a certain multi-objective optimization problem is given, where each component of the objective function and constraints is an implicitly defined real-valued function. Constraint C is satisfied for solution x if $C(x) = 0$, otherwise $C(x)$ shows how much the constraint is violated.

When modeling for components of a vector x, it is necessary to define the type of variable (boolean, integer, continuous, or some enumeration) and the bounds within which the variable can be changed. Additionally, some hints about the structure of the components of the objective function and constraints may be provided.

The work compares the solver with the MOEA Framework. Testing was carried out using classical problems to evaluate the effectiveness of algorithms and linear programming problems.

Keywords: multi-criteria optimization, implicitly defined function, approximate algorithm.

Maria Barkova[1], **Alexander Strekalovsky**[2]

On Solving Nonconvex Optimization Problem with Quadratic Data

[1] Matrosov Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences

[2] Matrosov Institute for System Dynamics and Control Theory SB RAS

We address a nonconvex quadratically constrained optimization problem with DC functions. Using the Exact Penalization theory, the original problem was reduced to a problem without constraints. To solve it, we use the Global Search Theory, which consists of two basic stages: (i) a special local search method (LSM) and (ii) procedures of escaping from critical points based on Global Optimality Conditions. In order to produce critical points, the special local search method was developed. It combines a consecutive solution of partially linearized problems and a procedure of penalty parameter changing. The effectiveness of the methodology is demonstrated by a large field of computational experiments. Besides, using special technique for generating test examples, the large series of high dimensional test problems with known global and local solutions was constructed. The results of numerical testing demonstrate considerable computational effectiveness of proposed approach, compared with other software for solving nonconvex quadratic problems with inequality constraints.

The research was supported by RSF (project No 24-41-03004).

Keywords: nonconvex quadratic problem, inequality constraints, DC functions, exact penalty, local search, global search.

George (Givi) Bolotashvili

FEATURES OF FACET CUTS WHEN SOLVING THE LINEAR ORDERING PROBLEM

When solving a linear ordering problem as an integer linear programming problem with an initial relaxation polytope, it matters which facets we use to cut off for the resulting non-integer vertex. In this paper, we consider a specific non-integer point and prove that this point is a non-integer vertex of the initial relaxation polytope. The special approach is that for this non-integer vertex in initial relaxation polytope we construct adjacent integer vertices and with the help of these integer vertices we uniquely find all the already known facet cuts. Note also that in this way obtained facet cuts play a decisive role when solving the problem.

Keywords: NP hard problem, linear ordering problem, polytope, non-integer vertex, facet.

Anton Bondarev

OPTIMIZATION IN HYBRID CONTROLLED SYSTEMS WITH PIECEWISE-SMOOTH SYSTEMS' DYNAMICS

In this talk I make a survey of several recent results of my co-authors and me concerning optimal control of piecewise-smooth systems. In particular, there are novel types of long-run optimality, not yet covered in the literature. This includes but is not limited to, optimal pseudoequilibrium, hybrid crossing limit cycles, heteroclinic cycles and quasi-perdiodic optimal trajectories. I discuss structural properties of underlying optimal control problems necessary for the emergence of these types of solutions.

Keywords: piecewise-smooth systems, optimal control, structural properties.

Pavel Borisovsky[1]

Application of GPU computing to solving discrete optimization problems

[1] Sobolev Institute of Mathematics SB RAS

Parallel computing on graphic processors (GPUs) is getting more and more popular. Since NVIDIA released the CUDA development tool, it has become convenient to use GPUs for general-purpose computing and not just for graphics display tasks. A feature of a GPU is the presence of a large (hundreds and thousands) number of cores, which allows to significantly speed up the calculation, but requires to design special parallel algorithms. While the development of traditional processors (CPUs) has recently slowed down, the characteristics of the GPU (number of cores, memory size, power consumption, and cost) are improving rapidly. In this tutorial, we will learn the basics of GPU computing in CUDA and OpenCL and briefly review the pros and cons of using GPUs in various discrete optimization algorithms.

The research was supported by Russian Science Foundation grant N 22-71-10015.

Keywords: Metaheuristics, GPU, CUDA, OpenCL.

Vladimir Bure, Elena Parilina[1]**, Artem Sedakov**[1]

Stable coalition structures under set solutions

[1] Saint Petersburg State University

When coalitions of agents are formed, the stability of the structure is a desired and even necessary feature that prevents the coalitions from breaking apart. In a transferable utility case, the analysis of associated cooperative games is of particular importance due to the multiple ways in which coalitions' payoffs can be allocated. The existing literature on this topic has identified a number of approaches for addressing the stability of a coalition structure when a point solution to the associated cooperative games, such as the Shapley value, is taken as an allocation of coalitions' payoffs. However, there is a lack of

results involving set solutions, where the coalitions' payoffs can be allocated in multiple feasible ways. In this talk, we will examine several approaches for determining the stability of a coalition structure with respect to the core, the classical set solution of cooperative game theory.

Keywords: TU game, coalition structure, the core, stability.

Igor Bykadorov

International Trade under Monopolistic Competition: Local Comparative Statics of Market Equilibrium w.r.t. Trade Balance

We study a variation of the model of international trade under monopolistic competition of producers [1], [2]. The mathematical apparatus uses the ideas of [3], [4]. The utility of con-sumers is an additive separable function, transport (trade) costs are of "iceberg type", there is labor balance in each country, and free entry condition is fulfilled. The main difference from traditional setting is the absence of the trade balance assumption, i.e., the possibility of a non-zero trade balance. We obtain the local comparative statics of the market equilibrium with re-spect to trade balance, pay special attention to two important marginal cases of trade costs: free trade and autarky.

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Keywords: Dixit-Stiglitz-Krugman Model, Market Equilibrium, Trade Balance Saldo, Comparative Statics, Free Trade, Autarky.

Xujin Chen[1]

Undirected Networks Immune to the Informational Braess's Paradox

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The Informational Braess's paradox exposes a counterintuitive phenomenon that disclosing additional road segments to some selfish travelers results in increased travel times for these individuals. This paradox expands upon the classic Braess's Paradox by relaxing the assumption that all travelers possess identical and complete information about the network. In this presentation, we explore structural conditions that prevent the informational Braess paradox in undirected networks.

Keywords: undirected networks, Braess's Paradox, selfish strategy.

Yukun Cheng[1], **Xiaotie Deng**[2], **Qi Qi**[3], **Xiang Yan**[4]

Truthfulness of a Network Resource-Sharing Protocol

[1] Suzhou University of Science and Technology

[2] Peking University

[3] Renmin University of China

[4] Huawei

We consider a sharing economy over a network in which each vertex agent allocates resources to its neighbors in response to their contributions. General equilibrium theory can be applied here to solve the problem of deciding how to fairly and efficiently allocate resources among agents as resource sharing over the network can be modeled as a pure exchange economy. It is known that proportional sharing dynamics converges to a market equilibrium solution. We are particularly interested in proportional sharing dynamics as a mechanism for network resource sharing. Our focus is on the key issue in internet market design: whether an agent may manipulate its report of its own private information to gain more resources under this mechanism. This work establishes the first mathematical proof that such a practical distributed network resource-sharing protocol is truthful against the manipulative strategies of feasible weight misreporting and edge deletion applied both individually and together.

Keywords: resource exchange, market equilibrium, strategic behavior, mechanism design, truthfulness.

Julia Chirkova[1]

Potential Game in General Transport Network with Symmetric Externalities

[1] Karelian Research Centre of RAS

The paper considers a model of a general transport network and BPR linear delay functions with externalities. We consider the case where the impact of channel loads to the delay is pairwise symmetric. For this case, it is proven

that the game of traffic allocation among channels is potential, and the price of anarchy is limited by the value 4/3.

Keywords: Wardrop equilibrium, Optimal profile, Social costs, Price of Anarchy, Symmetric externalities, BPR-functions, General network.

Biswarup Das, Roman Rodionov, Maxim Prihodko

A new method based on Langevin dynamics to find points of global minima of non-differentiable functions

We propose a new method to locate points of global minima of a non-differentiable function (satisfying some mild regularity conditions) as follows: (i) We prove the existence of a differentiable transformation of the given non-differentiable function which has the same global minima as the original function and show how one can recover the points of global minima of the original function from knowing those of this transformed function. (ii) We prove that this transformed function has finite Gibbs distribution. Since Gibbs distribution assigns high probability mass around the points of global minima, sampling from such distribution will produce points of global minima of the transformed function. (iii) We devise techniques for sampling from this Gibbs distribution as follows: (a) We construct a smooth family of probability densities which converge on the space of measures to this Gibbs distribution by solving a suitable Kolmogorov-Fokker-Planck partial differential equation. (b) We show that the family of random variables coresponding to the above family of probability densities satisfies a suitable stochastic differential equation (sde) of Langevin type. (c) We show that the above stochastic process converges in distribution to a random variable which has the above Gibbs distribution as its density, as time increases. This gives us a discrete scheme to sample according to the above Gibbs distribution, which we test on a well-known test function and show that it outperforms one of the state of the art algorithm, ADAM.

Keywords: Global optimization, Non-differentiable functions, Langevin dynamics, Kolmogorov-Fokker-Planck equations.

Ivan Davydov[1], **Vsevolod Akentev**[2]

Greedy algorithms for the temporal bin packing problem with failure domain

[1] Sobolev Institute of Mathematics

[2] Novosibirsk State University

The main directions of development in cloud computing systems are aimed at increasing the efficiency of using server resources and the reliability of virtual machines (VMs), as well as reducing operating costs. A common goal in such

systems is to minimize the number of servers required to maintain a given set of VMs, satisfying server capacity constraints. This goal practically leads to the well-known temporal bin packing problem. This study explores a generalization of this problem by introducing a concept of fault domains, where specific sets of VMs must operate in independent domain zones. For this NP-hard problem, we developed a heuristic for calculating the upper bound, based on the First Fit algorithm. Using the structural properties of the new constraints, we propose several efficient VM sequencing algorithms for the First Fit algorithm. Preliminary computational experiments on a publicly available dataset showed that the proposed heuristic method is capable of efficiently solving both small and large scale problems in a short time.

Keywords: NUMA architecture, failure domain, cloud computing, temporal bin packing.

Ivan Davydov[1], **Aliya Gabdullina, Margarita Shevtsova, Dmitry Arkhipov**

Tabu Search for a service zone clustering problem

[1] Sobolev Institute of Mathematics

Network maintenance by service engineers (SE) involves a range of activities to ensure that the network is functioning optimally and providing reliable service to users. To optimize the network maintenance process, company need to find suitable places for service offices, decide how to distribute engineers between them, and find a partition of sites into office responsibility zones. These decisions should take into account multiple factors: routes from offices and sites, fair workload distribution, zone topologies, etc. Such a problem can be considered as a generalization of a well-known facility location problem with additional geometric and workload constraints.

In this study, we consider the following formulation of the network maintenance optimization problem. There is a set of sites, a set of potential service office locations, and a limited number of engineers. The site workload is defined by a vector of integers. The objective is to decide which offices should be open, distribute engineers between offices, and find a site partition. With respect to the referenced industrial scenarios, different types of zone topologies are considered. Two criteria are considered to be minimized: total traveling time between offices and sites in a related zone and minimal deviation of workload per worker. An original multi-stage heuristic, which includes greedy and Tabu-search approaches, is proposed. The algorithm includes geometry-based search steps to obtain solutions with star and convex zone topologies. Numerical experiments on industrial instances with up to 10000 sites and 20 offices demonstrate the efficiency of the proposed approach. The obtained results

outperformed the CPLEX solver and demonstrated algorithm scalability and obtained solution quality.

Keywords: service engineer, clustering, cellular networks, facility location, Tabu search.

Andrey Dobrynin[1]

ON EDGE CRITICAL 4-CHROMATIC KOESTER GRAPHS

[1] Sobolev Institute of Mathematics SB RAS

A Koester graph G is a simple 4-regular plane graph formed by the superposition of a set S of circles in the plane, no two of which are tangent and no three circles have a common point. Crossing points and arcs of S correspond to vertices and edges of G, respectively. A graph G is edge critical if the removal of any edge decreases its chromatic number. A 4-chromatic edge critical Koester graph of order 28 generated by intersection of six circles is presented. This improves an upper bound for the smallest order of such graphs. The previous upper bound was established by Gerhard Koester in 1984 by constructing a graph with 40 vertices.

Keywords: graph coloring, critical graph, Koester graph.

Ruslan Drachev, Artem Panin[1]**, Alexander Plyasunov**

POLYNOMIALLY SOLVABLE CASES OF A THRESHOLD STABILITY PROBLEM FOR COMPETITIVE PRICING

[1] Sobolev Institute of Mathematics

A sequential game of the Stackelberg type with three players is considered. The first player is a leader company. It assigns prices in its facilities for a homogeneous product. Next, a follower company sets prices in its facilities. After that, each client chooses a service facility by minimizing the purchase and transportation costs while taking into account his financial resources (budgets). Each company aims to maximize its profit from product sales. The goal of the game is to find such a feasible solution and such a maximum deviation from the clients' budgets that the leader's profit is not less than a given threshold.

Uniform and discriminatory pricing strategies are considered. In the uniform strategy, the same price is set in all facilities. In the discriminatory strategy, the company determines its service price for each client in each facility.

It is proved that the problem is polynomially solvable under pricing strategies described above. Corresponding polynomial algorithms are proposed.

The work was supported by the Russian Science Foundation (project 23-21-00424).

Keywords: Stackelberg game, threshold stability problem, competitive pricing, polynomial algorithms.

Roman Emelyanov, Andrey Tikhomirov, Aleksandr Beznosikov[1], Alexander Gasnikov

Extragradient Sliding for Composite Non-Monotone Variational Inequalities

[1] Moscow Institute of Physics and Technology

Variational inequalities offer a versatile and straightforward approach to analyzing a broad range of equilibrium problems in both theoretical and practical fields. In this paper, we consider a composite generally non-monotone variational inequality represented as a sum of L_q-Lipschitz monotone and L_p-Lipschitz generally non-monotone operators. We applied a special sliding version of the classical ExtraGradient method to this problem and obtain better convergence results. In particular, to achieve ε-accuracy of the solution, the oracle complexity of the non-monotone operator Q for our algorithm is $\mathcal{O}\left(L_p^2/\varepsilon^2\right)$ in contrast to the basic ExtraGradient algorithm with $\mathcal{O}\left((L_p + L_q)^2/\varepsilon^2\right)$. The results of numerical experiments confirm the theoretical findings and show the superiority of the proposed method.

Keywords: Variational inequality, ExtraGradient, Composite problem, Sliding, Minty assumption.

Anton Eremeev[1], Valentin Topchii

Heavy-Tailed Mutation with a Regularly Varying Constraint on the Distribution Function of Its Rate

[1] Dostoevsky Omsk State University

We study the heavy-tailed mutation operator proposed by Doerr, Le, Makhmara, and Nguyen (GECCO 2017). The power-law assumption of mutation rate is generalizeed using regularly varying constraint on the distribution function of mutation rate. It is shown that, on the OneMax function class, the expected runtime of evolutionary algorithms with this generalized version of mutation is linear in the problem dimension.

Keywords: Evolutionary algorithms, Regularly varying function, Heavy-tailed mutation, Runtime.

Vladimir Erokhin[1]

A method for solving a dual pair of linear programming problems based on the fast Fejér type algorithm for finding a nonnega-

TIVE SOLUTION TO A SYSTEM OF LINEAR ALGEBRAIC EQUATIONS

[1] A.F. Mozhaisky Military Space Academy

The work is in line with the research of Fejér type iterative algorithms, the foundations of which were laid and developed in the works of I.I. Eremin, V.V. Vasin, L.D. Popov, E.A. Berdnikova, I.M. Sokolinskaya, L.B. Sokolinsky, E.A. Nurminski. The main instrumental result is a new variant of the Fejér type mapping, which includes a composition of the operation of orthogonal projection of a vector into the linear subspace of solutions of a system of linear algebraic equations (by means of its multiplication by the corresponding matrix-projector) and the operation of projection of a vector onto a non-negative orthant, but not by means of the traditional positive cut operation, but by means of the element-by-element operation of calculation of the absolute value. The global linear convergence of the above mapping is rigorously proved and its constant of asymptotics is estimated. Computational experiments demonstrate a significantly faster convergence of the studied mapping compared to the mapping using a positive cute operation. The next result is a Fejér type mapping for finding a solution to a system of linear inequalities. It is obtained from the previous mapping by introducing additional non-negative (slack) variables that turn inequalities into equalities. When realizing the above mapping with the help of identities of block matrices pseudo-inversion (one of the blocks of is a unit matrix), it is possible to avoid the growth of the dimension of the pseudoinverse matrix involved in the construction of the design matrix - its dimension remains the same as in the first mapping. The third result is a Fejér type mapping for finding a solution to a pair of mutually dual linear programming (LP) problems (in canonical and basic forms). It is obtained from the first two mappings using duality theory. In this case, the projection of the "long" vector composed of the solution vector of the primal problem, the solution vector of the dual problem and slack variables onto the linear subspace of solutions of the "large" system of linear algebraic equations can be realized with the help of matrices whose sizes do not exceed the sizes of the constraint matrix of the primal LP problem. This is achieved by using the identities of block matrices pseudo-inversion, the Sherman-Morrison-Woodbury identity and a fragment of the Greville's algorithm. For the described method of solving a dual pair of linear programming problems, computational experiments are in progress at the time of writing theses of this report. Preliminary results of the work were discussed on November 2, 2023 at the "Workshop on Optimization, Machine Learning and Artificial Intelligence "OandML" of St. Petersburg State University: http://oml.cmlaboratory.com/reps23.shtml#0309.

Keywords: Linear programming, Fejér type methods.

Maxim Ershov[1], **Albert Voroshilov**[1]

UCB strategy for batch data processing on an unknown horizon

[1] Yaroslav-the-Wise Novgorod State University

We consider the application of batch data processing and UCB strategy in the two-armed bandit problem. Batch processing implies that data is processed in batches, and the total income shows the count of successfully processed data. By virtue of the central limit theorem, incomes will have a normal distribution, as in the two-armed bandit problem. With a large count of data, determining the optimal strategy takes a long time. To solve this problem, we suggest using regression analysis. Using regression analysis, we calculated how the maximum normalized regrets will change if the count of data varies in the range from 5 thousand to 100 thousand. We propose to use the obtained dependencies on an unknown control horizon. The article presents numerical results and shows the choice of a strategy based on the dependencies obtained. We used the Monte Carlo method to simulate control in a random environment.

Keywords: two-armed bandit problem, UCB Strategy, batch processing, regression analysis.

Adil Erzin[1], **Maxim Anikeev**

Energy-efficient regular strip covering with fixed-size identical sectors

[1] Sobolev Institute of Mathematics

We consider the problem of covering a strip with identical sectors of given size in order to minimize the coverage density. Early in the literature, researchers considered the problems of constructing strip covers with circles, ellipses, and sectors of minimum density by proposing various covering models and determining the optimal values of the parameters of the covering figures for each model. In this paper, we consider identical sectors as covering figures and specify their parameters instead of optimizing them. We propose several new covering models and found a formula for calculating density for each model. Unfortunately, these formulas are quite cumbersome, which did not allow us to compare the densities of all covers analytically. Therefore, we defined some properties programmatically. As a result, a minimum density coverage model was determined for different values of the sector's parameters.

Keywords: Regular strip covering, Density minimization.

Adil Erzin[1], **Elizabeth Voronchikhina**

Covering mandatory and optional points in a strip with identical circles

[1] Sobolev Institute of Mathematics

There are two well-known problems of covering points with identical circles. In both problems there are a set of points arbitrarily located on the plane. In the first problem, it is necessary to cover all points with a minimum number of identical circles. In the second problem, it is required to cover the maximum number of points with a given number of identical circles. Both problems are NP-hard.

This paper considers a problem different from those mentioned above. The set of points is splitted into two subsets of mandatory and optional points. In turn, the set of mandatory points is divided into disjoint subsets (clusters). It is required to cover each cluster of mandatory points with one unit circle in such a way as to maximize the number of covered optional points. The complexity status of the problem is unknown. Preliminary results were previously obtained for it, but have not been published. This paper considers a special case of the problem in which all points are in a strip. Sufficient conditions are found for the existence of an optimal order-preserving cover, which is constructed in polynomial time using a dynamic programming algorithm.

Keywords: Mandatory and optional points, Optimal unit disk covering.

Adil Erzin[1], **Anzhela Shadrina**

Optimal placement of mobile sensors for distance-constrained line routing problem

[1] Sobolev Institute of Mathematics

A line segment (barrier) is specified on the plane, as well as the location of the depots. Each sensor can travel a limited-length path, starting and ending at its depot. The part of the barrier along which the sensor moved is *covered* by this sensor. It is necessary to determine the number of sensors (drones) in each depot in order to cover the entire barrier using a minimal number of drones (problem MinNum), or to minimize the maximum distance traveled by each drone (problem MinMax), or to minimize the total length of paths traveled by all drones (problem MinSum).

Previously, the problem MinNum of covering a barrier using minimal number of drones (one drone in each depot) was considered. In the problem considered in this paper, the solution is the number of drones in each depot, as well as the trajectory of each drone. We propose algorithms for solving the problem for all three criteria mentioned above.

Keywords: Barrier covering, Drones, Limited energy, Optimization.

Evgeny Goncharov[1]

A greedy algorithm for the resource-constrained project scheduling problem

[1] Sobolev institute of mathematics

The resource-constrained project scheduling problem (RCPSP) is a general scheduling problem that includes precedence and resource constraints. Activities preemptions are not allowed. Resources are renewable and there is a unique way to perform the activities. The problem with renewable resources is NP-hard in the strong sense. We propose a new deterministic greedy algorithm. It is based on heuristics that use information obtained from an relaxing problem. The algorithm is tested with standard data sets given by Kolisch library PSPLIB for J60, J90 and J120 and found to be performing well.

Keywords: Project management, resource-constrained project scheduling problem, renewable resources, Greedy algorithm, PSPLIB.

Aigul Fabarisova, Vadim Kartak

Multi-objective Optimization Approach for Polygon Coverage with Simple Shapes

The paper addresses a challenging problem related to full coverage path planning of car-like cleaning robots for cleaning areas of complex configurations. The approach involves dividing the given area, defined by a polygon with holes, into simpler shapes (e.g., rectangles, trapezoids) for which effective path planning methods are known. The coverage optimization problem arises where the objective is to minimize the total area of these shapes while practically covering the entire polygon. This problem is considered as a multi-objective optimization problem, with parameters such as the robot's turning radius, cleaning brush width, maximum shape area, and allowable angles. The solution approach consists of two stages. First, generating a set of potential candidates – shapes that meet the specified constraints. Here, two methods for generating discussed in detail. An original approach is suggested to ensure their coverage of the entire polygon. This involves creating a grid of base points that uniformly cover the entire polygon and using these points to generate candidates. This set of candidates is then consolidated to form the final set of candidates. Second stage is selecting an optimal solution from this set based on given criteria. Here, a linear integer programming model is proposed. The computational experiments suggest that a random grid generation algorithm often yields better quality partitions compared to a regular grid. The algorithm's overall computational time typically does not exceed 10 seconds, contributing to its practical utility in real-world applications.

Keywords: Multi-objective optimization, Coverage problem, Polygon Coverage.

Dmitrii Farladanskii[1], **Polina Kononova**[2]

Hybrid iterated local search algorithm for vehicle routing problem with traffic congestion

[1] Novosibirsk State University

[2] Sobolev Institute of Mathematics

We consider a family of rich vehicle routing problems (RVRP) which have the particularity to combine a heterogeneous fleet with other attributes. Modifying the Hybrid iterated local search algorithm with variable neighborhood descent and a set partitioning formulation (HILS-RVRP), we want to effectively solve more realistic problem combinations. It was decided to leave the following attributes: pickup and delivery, multiple depots, site dependency, open routes, duration limits, time windows, asymmetric distances and traffic congestion (or speed limits). Traffic congestion is realized using several matrices (12 or 24 in total) for 30 minutes or 1 hour period. Each describes coefficients for the vehicle speed on exact distances. The algorithm has a modular structure that allows you to select only the necessary attributes and thereby save computation time. Comparative characteristics and tests of the universal algorithm will be presented at the conference.

Keywords: variable neighborhood descent, set partitioning, pickup and delivery, multiple depots, site dependency, open routes, duration limits, time windows, asymmetric distances.

Zulfiya Gabidullina[1]

Assessing the Perron-Frobenius Root of Symmetric Positive Semidefinite Matrices by the Adaptive Steepest Descent Method

[1] Kazan federal university

We discuss the maximum eigenvalue problem which is fundamental in many cutting-edge research fields. We provide the necessary theoretical background required for applying the fully adaptive steepest descent method (or ASDM) to estimate the Perron-Frobenius root of symmetric positive semidefinite matrices. We reduce the problem of assessing the Perron-Frobenius root of a certain matrix to the problem of unconstrained optimization of the quadratic function associated with this matrix. We experimentally investigated the ability of ASDM to approximate the Perron-Frobenius root and carry out a comparative analysis of the obtained computational results with some others presented earlier in the literature. This study also provides some insight into the choice of parameters, which are computationally important, for ASDM. The study revealed that ASDM is suitable for estimating the Perron-Frobenius root of

matrices regardless of whether or not their elements are positive and regardless of the dimension of these matrices.

Keywords: adaptive steepest descent method, matrix norm, Perron-Frobenius root, spectral radius, dominant eigenvalue.

Mikhail Gladkikh[1], **Anna Melnik**[1], **Nelli Korotkova**

LAST MILE ROUTING FOR LTL SHIPMENTS

[1] BIA - Technologies

In this article we consider a problem of last mile routing for LTL shipments based on practical experience in developing a route planning system, that processes over 100,000 transportation tasks daily for 300 cities. The system incorporates a risk model, which leads away from optimal solutions towards maximum route stability. Experimental results from Homberger tests show that our algorithm has small deviations from known best solutions.

Keywords: Practical Vehicle Routing Problem, Time windows, Metaheuristics, Transportation, Less-than-truckload.

Aleksey Glebov

LOWER BOUNDS FOR GRUNDY INDEX OF SOME GRAPHS

The Grundy number $Gr(G)$ of a graph G is the maximum number of colours needed for greedy (proper) colouring vertices of G. Similarly, the Grundy index $Gr'(G)$ of a multigraph G is the maximum number of colours needed for greedy (proper) colouring edges of G. The concept of Grundy number has been actively studied in the literature lately. However, only one paper (L. Anderson et al., 2017) has been published about the Grundy index in which the authors computed $Gr'(G)$ for some specific graphs such as a complete graph, a complete bipartite graph, and some types of grids.

We present lower bounds for the Grundy index of some classes of graphs and multigraphs in terms of the minimum edge degree $d'(G)$ of a multigraph G. In particular, we prove that $Gr'(G) \geq d'(G)$ for any triangle free graph G and that $Gr'(G) \geq d'(G) + 1$ for any multigraph G with girth at least 5.

The research was performed according to the Government research assignment for IM SB RAS, project FWNF-2022-0017.

Keywords: Edge colouring, Greedy colouring, Grundy number, Grundy index, Minimum edge degree.

Hamidreza Golmohammadi

TOTAL COALITIONS IN CUBIC GRAPHS

A total coalition in a graph $G = (V, E)$ consists of two disjoint sets of vertices V_1 and V_2, neither of which is a total dominating set but whose union $V_1 \cup V_2$, is a total dominating set. A total coalition partition in a graph G of order $n = |V|$ is a vertex partition $\tau = \{V_1, V_2, ..., V_k\}$ such that every set $V_i \in \tau$ is not a total dominating set but forms a total coalition with another set $V_j \in \tau$ which is not a total dominating set. The total coalition number $TC(G)$ equals the maximum order k of a total coalition partition of G.

Keywords: Coalition; Total coalition; Cubic graphs; Petersen graph.

Shengjie Gong[1], Lingxiao Huang[2], Shuangping Huang, Yuyi Wang, Zhiqi Wang, Tao Xiao, Xiang Yan[3], Chunxue Yang

A Unified Framework of Multi-Stage Multi-Winner Voting: An Axiomatic Exploration

[1] South China University of Technology

[2] Nanjing University

[3] Huawei

Multi-winner voting plays a crucial role in selecting representative committees based on voter preferences. Previous research has predominantly focused on single-stage voting rules, which are susceptible to manipulation during preference collection. In order to mitigate manipulation and increase the cost associated with it, we propose the introduction of multiple stages in the voting procedure, leading to the development of a unified framework of multi-stage multi-winner voting rules. To shed light on this framework of voting methods, we conduct an axiomatic study, establishing provable conditions for achieving desired axioms within our model. Our theoretical findings can serve as a guide for the selection of appropriate multi-stage multi-winner voting rules.

Keywords: Multi-stage voting, Multi-winner voting, Axiomatic exploration.

Aleksandra Grinikh

The n-Person Prisoner's Dilemma on a Hypergraph.

N-Person "prisoner's dilemma" on a hypergraph is considered. In this new approach, players are represented as hypergraph nodes. This model is a modification of the classic problem of the game theory. Each hyperedge constitutes the simultaneous game between its nodes as players of this game. So, games are played between the participants of social communuties. The player's payoff function depends on the behaviour of closely located players on the hypergraph. Therefore, they play the games simultaneously on each hyperedge of the hypergraph. In this study, the non-cooperative game is considered and a payoff function is introduced, that is defined taking into account that players

can choose only one possible strategy for all hyperedges they belong to. In addition, we have proposed a repeated version of the model. Thus, it allows to analyse the cycle-consistent solution for the non-cooperative representation of the game. This work is a development of results published in Grinikh, A. L., Petrosyan, L. A. "An effective punishment for an n-person prisoner's dilemma on a network" (2021).

Keywords: n-person prisoner's dilemma, hypergraph, hyperedge, payoff function.

Zhenfei Guo[1], **Xi Chen, Hao Yang, Yun Dong**

Enhanced NSGA-II: Mitigating Limitations in Solution Set Size and Non-Dominant Solution Spacing for Wing Design Optimization

[1] National Frontiers Science Center for Industrial Intelligence and Systems Optimization, Northeastern University, China

When utilizing intelligent algorithms to optimize the thickness of wing skin for maximum stiffness in structural design, it is observed that certain areas of the skin thickness exhibit segmented nonlinear effects on wing stiffness, influenced by the geometry and material properties of the wing. This presents a challenge as a multi-objective optimization problem with discontinuous nonlinear constraints. Conventional NSGA-II algorithms struggle with such discontinuities, resulting in limited solution sets and wide fitness intervals among non-dominated solutions To address these limitations, this study enhances the NSGA-II optimization algorithm by introducing a local search mechanism. This enhancement expands the algorithm's coverage of the solution space, improves its exploration capabilities. Additionally, the selection strategy is adjusted to optimize the retention of excellent solutions while promoting diversity. Furthermore, a reward -punishment mechanism is incorporated into the calculation of fitness function values, motivating the algorithm to explore more regions of the solution space and generate a broader range of solutions. Following these improvements, the average number of solution spaces increases by approximately 20%. Moreover, the spacing between non-dominated solutions is significantly reduced, leading to the generation of more feasible solutions. This expanded solution set offers designers greater flexibility and options when approaching wing structure design.

Keywords: multi-objective optimization, structural design, reward-punishment mechanism, local search, discontinuous nonlinear constraints.

Nikolai Guschinsky[1]

Models and Methods for Optimization of Electric Public Transport Systems

[1] United Institute of Informatics Problems, Minsk, Belarus

The transition towards sustainable cities becomes an increasingly pressing issue because of the growing awareness about the climate change. One of the critical transition directions is to reduce the greenhouse gas emissions generated by transportation. One of efficient answers to this challenge is the implementation of electric public transportation systems. In order to achieve the best impact, it is imperative to design the infrastructure and the network of the public transportation in an optimized way. The following problems concerning strategic, tactical and operational aspects of the electric bus planning process and scheduling are discussed: a) investment of electric bus fleet and charging infrastructure; b) design of charging infrastructure; c) the electric vehicle scheduling; d) the charging scheduling problem. The models and optimization methods for different charging technologies are considered:

– slow plug-in chargers installed at bus depots;

– fast plug-in or pantograph chargers installed at terminals of bus lines or at bus stops;

– overhead contact lines or inductive (wireless) chargers that are used to recharge buses during driving;

– battery swapping.

Keywords: network of the public transportation, electric bus scheduling, charging infrastructure.

Vasily Gusev, Alexander Nesterov, Mikhail Reshetov, Alex Suzdaltsev

The Existence of a Pure-strategy Equilibrium in a Discrete Ponds Dilemma

In a variety of economic situations discrete agents choose one resource among several available resources and, once admitted to the resource of choice, divide it among fellow agents admitted there. The amount of the resource an agent gets is proportional to her relative ability to acquire this particular resource, what we refer to as an agent's weight at the resource. The relevant applications include students self-selecting into colleges, politicians self-selecting into races, and athletes self-selecting into teams. We find that this game has a pure-strategy Nash equilibrium in at least three special cases: 1) when agents have the same weight at each resource, 2) when all resources are the same, 3) when there are only two resources. We also show that this game always has an approximate Nash equilibrium when the number of players is large. Existence in the general case remains an open problem.

Keywords: congestion games, potential games, pure Nash equilibrium, sort-

ing into contests, college admissions.

Vugar Hashimov[1], **Kamil Aida-zade**[1]

Optimization of the measurement points movement in one problem of synthesis of temperature control of a furnace for heating the rods

[1] Institute of Control Systems of the Ministry of Science and Education of Republic of Azerbaijan

The problem of optimal feedback control of the furnace rod heating process is studied. Heating is carried out by the heat supplied to the furnace. The temperature of the heat supplied to the furnace depends on the results of heat measurements at the points of the rod. Measurements of the state of the heating process are carried out using sensors moving along the rod, the speed of which is determined based on the results of current measurements. The heating process is described by a boundary value problem with respect to a one-dimensional differential equation of parabolic type. The motion of sources is described by nonlinear differential equations with ordinary derivatives. For feedback, linear dependences of the control actions are used - heat temperature and the speed of movement of the state measurement points on the measured states of the process on the rod. The coefficients involved in these dependencies are feedback parameters. Thus, the problem of optimal control synthesis for the process under consideration is reduced to optimization of a finite-dimensional vector of feedback parameters. The corresponding problem belongs to the class of parametric problems of optimal control of objects with distributed parameters. To numerically solve this problem, formulas for the gradient components of the objective functional in the space of feedback parameters are obtained. The resulting formula was used to construct algorithms for numerical methods of first-order optimal control. Namely, the method of projection of the penalty functional was used. The projection was carried out on positional constraints, the penalty method was used to consider phase constraints.

The results of computer experiments obtained while solving the test problem are presented.

Keywords: rod heating, feedback control, moving sensor, feedback parameters.

Roland Hildebrand[1]

Tuning Methods for Minimization of Self-concordant Functions with Optimal Control

[1] Moscow Institute of Physics and Technology, Dolgoprudny, Russia

In the last decade the usage of optimization methods for performance estimation and tuning of other optimization methods has became fashionable. For instance, semi-definite programming allows to analyze the behaviour of accelerated gradient descent methods at minimizing functions of different regularity. Detecting the worst-case performance with respect to the minimized function and maximizing this performance with respect to the parameters of the method allows to obtain the best parameter set. Here we show how to perform a similar analysis of the Newton method when minimizing self- concordant functions. This task arises as a subproblem in more complex structured convex optimization problems such as semi-definite programming inside path-following methods. The appropriate framework for optimizing the parameters, in particular, the step length and the search direction, is optimal control theory. We present a general technique to use the Pontryagin maximum principle in the analysis of the Newton step on a self-concordant function and specialize it in several case studies.

Keywords: convex optimization, self-concordant function, semidefinite programming.

Shiyan Huang, Lixin Tang, Jian Wu

Functional data prediction with time lag

Many large systems, such as manufacturing plants or equipment aging in service, generate a number of time series. Understanding the temporal relationships between the data is important for prediction and monitoring. In practical scenarios there exists a situation where the dependent variable is only correlated with the time series within a certain moment in the past. To address this problem, we propose a new approach for the prediction that includes a time-lag penalty. This task is formulated as a functional data prediction problem, where the time series and the predicted coefficients are represented as a basis function expansion and the predictions of some of the covariates are sparsely contracted to zero. We develop a convex model and solve it with convex optimization methods. Experiments demonstrate that the functional time-lag prediction model has good prediction results in systems with time-lag effects, indicating that the model proposed in this paper is practical and effective.

Keywords: time series, time-lag, functional data, convex optimization.

Kuankuan Huang[1], Yu Yue, Yueyu Liu, Xuedong Liang

Decision Analysis of Military Supply Chain Based on Stackelberg Game Model

[1] Sichuan University

The military supply chain assumes a paramount role in national security and defense infrastructure development, entailing substantial economic implications. Scientifically informed managerial decisions serve as indispensable mechanisms for bolstering the operational efficiency and efficacy of the military supply chain, thereby posing a pragmatic challenge to contemporary decision-makers. This paper introduces the training task volume of military units as an indicator for measuring both the demand of the military and the efficiency of the supply chain for the first time. By constructing a Stackelberg game model, the decision-making behaviors of various members in the military supply chain under different leaderships are studied to explore the influence of leadership on optimal decision-making by each member in the supply chain. The research indicates that leadership dominance significantly affects the profits of members in the military supply chain. A centralized decision-making model is more effective than a decentralized decision-making model in maximizing the overall profits of the military supply chain, providing valuable insights for optimizing management and making scientifically informed decisions in the military supply chain.

Keywords: Decision analysis, Stackelberg game, Military supply chain.

Danis Ibragimov[1]

On the Method for Refining A Priori Estimates of the Objective Function in the Speed-in-Action Problem for a Linear Discrete-Time System

[1] Moscow Aviation Institute (National Research University)

A two-dimensional linear discrete-time system with bounded control is considered. The speed-in-action problem for a given system is studied, in particular, the issues of a priori estimation of the minimum number of steps required to transfer the system from the initial state to the origin of coordinates are studied. The method for systems with a diagonalizable matrix is proposed for constructing effective estimates of the optimal value of the objective function in the speed-in-action problem as a function of the initial state. Sufficient conditions are formulated under which these estimates cannot be improved. Numerical calculations are presented.

Keywords: Linear Discrete-Time System, Speed-in-Action Problem, Optimal Control, A Priori Estimates of the Optimal Value of the Objective Function, Estimation Refinement.

Igor Konnov[1]

Convergence Properties of a Primal-Dual Method for Multi-agent Optimization Problems

[1] Kazan Federal University

We first describe a modified primal-dual method that uses variable metric matrices for general convex optimization problems with changing affine constraints. We establish convergence of the method under general assumptions that uses variable metric matrices at each iteration. This approach yields new opportunities for control of the parameters according to the constraints changes. In case of the multi-agent optimization problems the method can be adjusted to the changing communication topology and enables the agents to choose the parameters separately of each other. We also show that the primal-dual methods possess stable convergence properties under different assumptions. Significant examples of applications are also given.

Keywords: Convex optimization, changing constraints, primal-dual method, variable metric, constrained multi-agent optimization, stable convergence.

Artem Il'ev[1], **Victor Il'ev**[2]

Clustering Complexity and an Approximation Algorithm for a Version of the Cluster Editing Problem

[1] Sobolev Institute of Mathematics SB RAS

[2] Dostoevsky Omsk State University

In graph clustering problems, one has to partition the vertex set of a given undirected graph into pairwise disjoint subsets (clusters). Vertices of the graph correspond to some objects, edges connect the pairs of similar objects. In cluster editing (CE) problems the goal is to find a nearest to a given graph $G = (V, E)$ cluster graph, i.e., a graph on the same vertex set V each connected component of which is a complete graph. The distance between graphs is understood as the number of their non-coinciding edges. The distance between a graph G and a nearest to G cluster graph is called clustering complexity of G. We consider a version of CE problem in which the size of each cluster is bounded from above by a positive integer s. This problem is NP-hard for any fixed $s > 2$. In 2015, Puleo and Milenkovic proposed a 6-approximation algorithm for this problem. For the version of the problem with $s = 5$ we propose a polynomial-time approximation algorithm with better performance guarantee and prove an upper bound on clustering complexity of a graph that is better than earlier known one.

Keywords: graph cluster editing, approximation algorithm, performance guarantee, clustering complexity.

Nikolay Ivanov, Oleg Khamisov

Optimization of Line Differential Protection Settings in Power Systems Using Dual-Slope Restraint Characteristics

This paper delves into optimization of line differential protection settings in power systems. The differential relay monitors current at both ends of a transmission line and issues trip signals in the presence of significant imbalances during internal faults. The differential relay incorporates specific dual slope restraint characteristics, whose slopes determine the relay's sensitivity to internal faults while mitigating the risk of unnecessary tripping during normal and external fault events. The biggest challenge for line differential protection is to ensure its correct operation during saturation of currents transformers that results in distorted measurements for differential relay. Traditionally, relay settings are calculated based on severe cases, lacking adaptability to network changes. The paper proposes a dynamic and adaptive approach, employing a digital twin for fault scenario simulations. To capture the dynamic behavior of the system, the Dommel's method, a numerical integration technique based on the trapezoidal rule, is employed. The optimization process aims to determine optimal coefficients for relay characteristics, enhancing the sensitivity of the protection scheme to internal faults while mitigating the risk of unnecessary tripping during normal and external fault events. The objective function evaluates the performance of the dual-slope restraint characteristic. It considers the number of false trips and the area under the piecewise linear function as two optimization goals. The number of false trips is minimized to ensure correct relay operation during internal and external faults. The area under the curve is minimized to enhance the relay's sensitivity and responsiveness to fault conditions. Overall, this methodology offers a robust framework for improving the performance and reliability of line differential protection, providing a responsive solution to evolving power system dynamics.

Keywords: Power systems, Line differential protection, Dommel's method.

Ilya Ivantsov, Makar Sidorov

Object-oriented approach in operational planning and scheduling problems

The MILP-problems related with operational planning and scheduling usually are quite large and complicated. Traditional representation of such problems is a list of variables and constraints (like it implemented in MiniZinc open-source constraint modeling language). But this approach leads to problems such as difficulty in expanding model, debugging, covering it by tests. It is impossible to create a good program architecture in this way, which will be convenient for maintaining the model over a long period of time. So we in-

troduce an object-oriented approach to decompose such a huge models with Pyomo (Python-based, open-source optimization modeling language). We will follow the basic principles of OOP: abstraction, decomposition, encapsulation, inheritance. This approach will provide us benefits, such as modularity, code maintainability, ability to reuse our code and ability to construct models for presolvers or heuristics by aggregating already defined submodels.

Keywords: mixed integer linear programming (MILP), object-oriented approach (OOP), Pyomo (open-source optimization modeling language), operational planning, scheduling.

Igor' Izmest'ev[1]

On a differential game of pursuit-retention

[1] Chelyabinsk State University

In Euclidean space we consider an antagonistic differential game of a given duration. Vectograms of players are n-dimensional balls, the radii of which are functions of time. The motion is constructed using polygonal lines. The goal of the first player is to lead the phase vector in the ring (a set determined by the condition that the norm of the phase vector belongs to a segment with positive ends) at some time moment, and then retain it there for a given time. In addition, the norm of the phase vector during the game process must be no less than a given number. The goal of the second player is the opposite. We have found the necessary and sufficient conditions under which the first player can guarantee the achievement of his goal, and constructed the corresponding controls of the players. The research was supported by a grant from the Russian Science Foundation no. 23-21-00539. https://rscf.ru/project/23-21-00539/.

Keywords: control, differential game, pursuit, retention.

Polina Kaidash[1]

Super Domination Polynomial of a Graph

[1] Novosibirsk State University

In this paper, a super domination polynomial of a simple graph $G = (V, E)$ of order $|V| = n$ is introduced as the polynomial $D_{sp}(G, x) = \sum_{i=\gamma_{sp}(G)}^{n} d_{sp}(G, i)x^i$, where $\gamma_{sp}(G)$ is the minimum cardinality of a super dominating set in G and $d_{sp}(G, i)$ is the number of super dominating sets S_{sp} of G of size i, and $\gamma_{sp}(G)$ is the super domination number of G. Some properties of $D_{sp}(G, x)$ and its coefficients for a given graph G are obtained. Furthermore, explicit formulas of the super domination polynomial of some families of graphs are presented.

Keywords: Dominating set, Super dominating set, Domination polynomial, Super domination polynomial.

Valeriy Kalyagin[1], **Ilya Kostylev**

ROBUSTNESS OF GRAPHICAL LASSO OPTIMIZATION ALGORITHM FOR LEARNING A GRAPHICAL MODEL

[1] HSE University

Problem of learning a graphical model (graphical model selection problem) consists of recovering a conditional dependence structure (concentration graph) from data given as a sample of observations from a random vector. Various algorithms to solve this problem are known. One class of algorithms is related with convex optimization problem with additional lasso regularization term. Such algorithms are called graphical lasso algorithms. Various properties and practical efficiency of graphical lasso algorithms were investigated in the literature. In the present paper we study sensitivity of uncertainty (level of error) of graphical lasso algorithms to the change of distribution of the random vector. This issue is not well studied yet. First, we show that uncertainty of the classical version of graphical lasso algorithm is very sensitive to the change of distribution. Next, we suggest simple modifications of this algorithm which are much more robust in the large class of distributions. Finally, we discuss a future development of the proposed approach.

Keywords: graphical model, concentration graph, uncertainty, graphical lasso optimization, robustness.

Alexander Kazakov[1], **Anna Lempert**[1], **Tran Tuan Viet**

A HEURISTIC ALGORITHM FOR COLLISION-FREE PATH PLANNING IN DYNAMIC ENVIRONMENT

[1] Matrosov Institute for System Dynamics and Control Theory of SB RAS

The report is devoted to collision-free path planning in a dynamic environment. As a distance between two points, we use the shortest time spent to achieve one point from another. We consider the infinite-dimensional routing problem, which arises when determining air, sea, and land routes with an infinite set of admissible trajectories. We propose a numerical algorithm that allows us to construct the fastest paths for one or several vehicles in the presence of permanent and moving obstacles. It is based on the optical-geometric approach, the analogy between the light propagation in an optically inhomogeneous medium and the minimization of the integral functional. Besides, we implement the curve-shortening flow method and compare the numerical results.

Keywords: path planning, optical-geometric approach, inhomogeneous medium,

moving obstacle.

Oleg Khamisov

Modified primal-dual algorithm for distributed optimization with coupling nonlinear constraints

This work is dedicated to distributed optimization problem with coupling nonlinear constraints. The problem consists of convex continuously differentiable objective function under smooth equality and inequality constants. It is assumed that the problem must be solved on multi-agent network with information exchange only between neighboring agents. We suggest to adopt primal-dual approach for this problem. The main effort was made towards overcoming difficulties of coupling constraints nonlinearity. We provide detailed description of the developed algorithm together with detailed numerical experiment for power system problems performed on a setup consisting of Real time Digital Simulator running physical power system connected to the PC with optimization routine.

Keywords: Distributed optimization, Nonlinear constraints, Rtds, Primal-dual algorithm.

Oleg Khamisov, Sergey Rozinov

Deep cuts in concave programming

The problem under consideration is global minimization of cancave function over a polytope. We present results of testing the algorithm based on constructing cutting planes through the best extension of the concave function over the polytope. Theoretical results providing convergence of the suggested algorithm to a global minimum are given. We also suggest a combination of the considered cuts with the general branch and bound scheme.

Keywords: concave programming, polytope, cutting plane.

Oleg Khamisov, Надежда Ульянова

Совместное планирование работы энергетических источников и управление потоками в энергосистемах

Рассматривается задача поиска оптимального режима работы энергетической системы, включающей источники и потребителей энергии, а также транзитные узлы. Произведённая суммарная нагрузка на каждом периоде времени всеми производителями покрывает потребности потребляющих станций. Требуется минимизировать суммарные издержки, которые несут источники для обеспечения необходимой нагрузки, а также затраты на транс-

портировку энергии. Издержки каждого источника включают переменную составляющую в виде кубической функции от объема производимой энергии и постоянную составляющую, направленную на содержание источника при нахождении в горячем резерве. Учитываются периоды ввода и вывода источника из рабочего состояния, в которые источники несут издержки, но энергию не вырабатывают. С применением бинарных переменных, определяющих состояние источников, рассматриваемая задача формулируется как задача частично целочисленного программирования. Решение задачи основано на построении аппроксимации сформулированной задачи задачами линейного частично целочисленного программирования путем замены кубической функции переменных издержек ее кусочно-линейными аппроксимациями. Предложен алгоритм решения задачи на основе предложенных аппроксимаций и выполнена его численная реализация. Проведен ряд тестовых расчетов.

Keywords: Энергетическая система, оптимизация, смешанное целочисленное программирование, линейные аппроксимации, численный алгоритм.

Vladimir Khandeev[1], **Sergey Neshchadim**[2]

Pseudo-polynomial algorithms for some problems of searching for the largest subsets

[1] Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences

[2] Novosibirsk State University

We consider several problems of finding the subsets with the largest minimal cardinality and limited scatter in a finite set of points in Euclidean space. For each cluster, the scatter is the sum of the distances (raised to a power) between the elements and the center of the cluster. Depending on the problem, the center can be defined in different ways - as a fixed point, as the centroid of the cluster, etc. We propose a general scheme for a pseudo-polynomial algorithm to solve such problems, and we also show how and in what time this scheme can be implemented for several types of centers.

Keywords: Euclidean space, Clustering, Max-min problem, NP-hardness, Bounded scatter, Pseudo-polynomial algorithm.

Yaroslav Kharchenko[1], **Alexander Kononov**[2]

A Learning-Augmented Algorithm for the Parking Permit Problem with Three Permit Types

[1] Novosibirsk State University

[2] Sobolev Institute of Mathematics

We consider the parking permit problem with three permit types. First, we prove the tight lower bound on the competitiveness of any deterministic online algorithm. Next, we present a learning-augmented algorithm and show its' consistency and robustness.

Keywords: learning-augmented, online algorithm, prediction.

Raisa Kim, Daniil Musatov[1]

ROADS BUILT IN A DEMOCRATIC WAY: VOTING AND ROUTING PARADOXES COMBINED

[1] Moscow Institute of Physics and Technology

Modern urban and regional infrastructure heavily relies upon the network of public-access automobile roads. Often construction of a new road is financed by general-purpose taxes. In this case the taxpayers would like to be better off after the road is built. The well-known Braess paradox shows that if the network is congested and routing decisions are decentralized, then building a new road may increase the spent time even for all road users. In this case farsighted users will not vote for such a construction even if it is free of payment.

We explore the cases when the situation is more complex. After a new road is built, some users become better off, the others become worse off. A user votes for or against the project depending on whether he becomes better off or worse off under the new equilibrium. We analyze several situations that can be viewed as paradoxes. In the first example after implementing the project the total time spent by all agents rises but a majority of agents votes for the project. In the second example the situation is the opposite: the project increases the total utility but a majority votes against it. In the third example we consider a society divided into three entities with separate budgets and show that they cannot reach a stable decision. Finally we discuss the notion of price of anarchy in this framework and prove some bounds on it.

Keywords: Transportation networks, Braess paradox, majority voting, coalitional stability, price of anarchy.

Yury Kochetov[1]

BLACK BOX OPTIMIZATION FOR BUSINESS APPLICATIONS

[1] Sobolev Institute of Mathematics, Novosibirsk, Russia

The black-box optimization models are characterized by lack of analytical forms for the constraints and the objectives of the problem. In the black-box methods, we need efficiently integrate known analytical part with explicitly unknown correlations obtained from the business simulation models. The direct use of classical global optimization methods is prohibitive due to the lack

of exact mathematical expressions. We cannot calculate the derivatives or subgradients. Moreover, the computational cost is high due to the simulations. Hence, we have to use the problem specific methods to optimize such black-box systems efficiently. Important applications stem from various disciplines: multi-echelon inventory systems, chemical and mechanical engineering, financial management, network topology design, and others. In this talk, we will discuss some directions in this area and theoretical bounds for global optimization methods. Successful cases for real-world applications will be presented.

Keywords: simulation model, mix-integer programming, multiobjective optimization.

Alexander Kolnogorov[1]

UCB Strategies in a Gaussian Two-Armed Bandit Problem

[1] Yaroslav-the-Wise Novgorod State University

We consider the two-armed bandit problem in the application to batch data processing if there are two alternative processing methods with different a priori unknown efficiencies, and income is understood as successfully processed data. It is necessary to determine a more effective method and ensure its preferential use. Batch processing means that the incomes in batches have Gaussian distributions with a priori unknown one-step mathematical expectations and variances. This corresponds to a situation when the number of processed data batches and their volumes are of moderate size. We use UCB strategies for control. To calculate the regret, a recursive dynamic programming equation is obtained. Using the properties of UCB strategies, this equation was presented in a more computationally convenient form and then presented in an invariant form with a control horizon equal to one. The invariant equation does not depend on the total number of processed data, but only on the number of batches into which the data is divided and on the number of internal packets for which the variance is estimated.

Keywords: two-armed bandit problem, batch processing, UCB strategies, Bayesian approach, invariant description.

Aleksei Kondratev

Veto core consistent voting rules

The proportional veto principle is the notion that a coalition of x percent of the voters should be able to block roughly x percent of the outcomes. This is in opposition to the majority principle, which holds that 51 percent of the voters should have all the decision power and the remaining 49 percent zero; or the utilitarian principle, which focuses selecting an outcome that is best on

average, even if that outcome is inadmissible for certain individuals or groups. Originating in public choice, the proportional veto principle found rich application in the theory of social multi-criteria evaluation and bilateral choice, but to date the family of voting rules which are consistent with this principle have not been subject to the same axiomatic scrutiny as majoritarian or positional approaches to voting. In this paper we address this gap by analysing two broad families of such rules, and six concrete examples, with respect to the properties of monotonicity, participation, and independence of unanimous losers.

Keywords: proportional veto, preference aggregation, rank aggregation.

Elena Konovalchikova, Vladimir Mazalov[1]

PRICING EQUILIBRIUM MODEL IN PUBLIC TRANSPORTATION MARKET

[1] Karelia research center

A game-theoretic model of pricing in the urban public transportation market is considered. It is assumed that the players in the model are transport companies serving urban public transport routes, and the distribution of passengers along the routes is carried out using the Hotelling specification. The study focuses on the Nash equilibrium in the pricing game of the market for transport services. The results of numerical modeling are presented using the example of the transport system in Petrozavodsk.

Keywords: Nash equilibrium, Public transportation market, Hotelling specification.

Elena Konstantinova[1,2]

RECENT PROGRESS IN DOMINATION THEORY

[1] China Three Gorges University, Yichang, China

[2] Sobolev Institute of Mathematics, Novosibirsk, Russia

In this talk, some recent results in domination theory are presented. In 2020, T. W. Haynes et al. introduced coalitions and coalition partitions based on dominating sets in graphs as a graph-theoretical model to describe political coalitions. The authors have been studied the property of this concept and suggested a list of open problems. In particular, it was suggested to study connected coalitions based on connected dominating sets. In this talk we focus on studying connected coalitions and their partitions in graphs with emphasising to polynomial-time algorithms determining whether the connected coalition number of a graph G of order n is either n or $n-1$. The talk is based on joint works with S. Alikhani, D. Bakhshesh, and H. Golmohammadi.

The research was supported by the Russian Science Foundation under grant no. 23-21-00459.

Keywords: domination theory, connected coalitions, dominating sets in graph.

Nikita Kosyanov

On finding optimal control in epidemic processes

All over the world there is a need to modify and adapt known epidemic models due to the increase in the level of urbanization of cities, as well as the development of medicine. We address the optimal control problem with compartment epidemic models (such as SIS, SIR, SIRD etc.) to find the best treatment and isolation strategies in a situation where an infectious disease is spreading. The main difficulty of solving optimal control problems in epidemic processes is that the controlled system is given by nonlinear differential equations. In addition, different modifications of epidemic models (e.g., models with vaccination) greatly complicate the search for optimal control in the problem. Until now, the search for optimal control in these problems has been based either on Pontryagin's maximum principle (which is applicable only in the case of a linear controlled system) or on dynamic and quadratic programmings. This algorithms based on classical convex optimization theory, but rather often applied for nonconvex problems. Thus, it is necessary to revise the approach to solving problems of epidemic processes control and use special methods based on nonconvex optimisation theory. The global optimality conditions in nonlinear optimisation problems can be taken as a basis to develope methods for finding optimal control in virus spreading problems.

Keywords: optimal control, compartment epidemic models, nonconvex optimization.

Matvei Kotov[1], **Alexander Treier, Ivan Buchinskiy**

On complexity of the problem of solving systems of tropical polynomial equations of degree two

[1] Sobolev Institute of Mathematics of SB RAS

In this paper, we investigate the computational complexity of the problem of solving a one-sided system of equations of degree two of a special form over the max-plus algebra. Also, we consider the asymptotic density of solvable systems of this form. Such systems have appeared during the analysis of some tropical cryptography protocols that were recently suggested. We show how this problem is related to the integer linear programming problem and prove that this problem is NP-complete. We show that the asymptotic density of solvable systems of this form with some restrictions on the coefficients, the number of variables, and the number of equations is 0. As a corollary, we prove that this problem (with some restrictions on the coefficients, the number of variables,

and the number of equations) is decidable generically in polynomial time.

Keywords: tropical algebra, generic-case complexity, asymptotic density, NP-completeness.

Roman Kozhevnikov[1], **Ivan Davydov**[2]

TABU SEARCH FOR THE (R—P)-CENTROID PROBLEM UNDER UNCERTAINTY

[1] Novosibirsk State University

[2] Sobolev Institute of Mathematics

We consider the (r—p)-centroid problem under an assumption of uncertainty in the input data. Two players, the leader and the follower, open facilities, striving to capture the largest market share. The leader opens p facilities, then the follower opens r facilities. Each customer chooses the nearest facility as his supplier and bring a particular income to the corresponding player. The aim is to choose p facilities of the leader in such a way as to maximize his market share. However, it is assumed that customers' purchasing power is not known exactly and can change after the decision made. This problem can be represented as a bilevel programming problem. In this work, we propose a local search approach based on a stochastic tabu search framework to tackle the problem. We provide the results of numerical experiments and compare the behavior of both players compared to the deterministic setting of the same problem.

Keywords: uncertain data, metaheuristic, centroid, bi-level programming.

Vladimir Krutikov, Elena Tovbis, Predrag Stanimirovic[1], **Lev Kazakovtsev**

CORRECTION OF METRIC MATRICES IN QUASI-NEWTON METHODS FROM THE PERSPECTIVE OF MACHINE LEARNING THEORY

[1] University of Nis, Faculty of Sciences and Mathematics

We consider correction of metric matrices in quasi-Newton methods from the perspective of machine learning theory. Based on the training information for estimating the matrix of the second derivatives of the function, a quality functional is formulated and minimized using gradient machine learning algorithms. In this way, it is possible to obtain the well-known formulas for updating metric matrices used in quasi-Newton methods. The learning algorithm for finding metric matrices performs minimization along a system of directions, the orthogonality of which determines the convergence rate of the learning process. The degree of orthogonality of learning vectors can be increased both by choosing a quasi-Newton method, and artificially, by using additional orthogonalization methods. It was shown theoretically, that the

orthogonality degree of learning vectors in the BFGS method is higher than in the DFP method, which determines the advantage of the BFGS. The work discusses some orthogonalization techniques. One of them is to include iterations with orthogonalization or exact one-dimensional descent. As a result, on quadratic functions it is theoretically possible to detect the cumulative effect of reducing the optimization space. Another way to increase the degree of orthogonality of learning vectors at the initial stages of the quasi-Newton method is a special choice of initial metric matrices. The stated theoretical positions are confirmed by a computational experiment on problems with a high degree of conditionality.

Keywords: convergence rate, machine learning, minimization algorithm, quasi-Newton method.

Vladimir Krutikov, Elena Tovbis, Predrag Stanimirovic[1], Lev Kazakovtsev

NEWTONIAN PROPERTY OF SUBGRADENT METHOD WITH OPTIMIZATION OF PARAMETERS OF RANK TWO CORRECTION OF METRIC MATRICES

[1] University of Nis, Faculty of Sciences and Mathematics

The work proves that under conditions of instability of the second derivatives of the function in the minimization region, the estimate of the convergence rate of Newton method is determined by the parameters of the irreducible part of the conditionality degree of the problem. These parameters represent the degree of difference in the eigenvalues of the matrices of the second derivatives in the coordinate system, where these differences are minimal, and the resulting estimate of the convergence rate subsequently acts as a standard. The paper studies the convergence rate of the relaxation subgradient method (RSM) with optimization of the parameters of two-rank correction of metric matrices on smooth strongly convex functions with a Lipschitz gradient without assumptions about the existence of second derivatives of the function. The considered RSM is similar in structure to quasi-Newton minimization methods. Unlike the latter, its metric matrix is not an approximation of the inverse matrix of second derivatives, but is adjusted in such a way that it enables one to find the descent direction that takes the method beyond a certain neighborhood of the current minimum as a result of one-dimensional minimization along it. This means that the metric matrix enables one to turn the current gradient into a direction that is gradient-consistent with the set of gradients of some neighborhood of the current minimum. Under broad assumptions on the parameters of transformations of metric matrices, an estimate of the convergence rate of the studied RSM and an estimate of its ability to exclude removable linear background are obtained. The obtained estimates turn out to be qualitatively

similar to estimates for Newton's method. In this case, the assumption of the existence of second derivatives of the function is not required. A computational experiment was carried out in which the quasi-Newton BFGS method and the subgradient method under study were compared on various types of smooth functions. The testing results indicate the effectiveness of the subgradient method in minimizing smooth functions with a high degree of conditionality of the problem and its ability to eliminate the linear background that worsens the convergence.

Keywords: convergence rate, minimization algorithm, quasi-Newton method, subgradient method.

Alexander Krylatov, Maksim Korol, Anastasiya Raevskaya

EQUILIBRATION OPERATORS FOR ACCURATE TRAFFIC FLOW ASSIGNMENT IN URBAN ROAD NETWORKS

The traffic assignment problem is known as an optimization problem with a non-linear objective function and linear constraints, which allows one to estimate traffic congestion in an urban road network. Very early ideas on solving the traffic assignment problem concerned the concept of the existence of an equilibration operator. Such an operator was assumed to return assignment patterns that provide equal travel times on used routes between an origin-destination pair. Despite attempts to express this operator mathematically the first successful implementation of the idea came from computational techniques. The concept appeared to be fruitful and easy to apply from a computational perspective. Recently, a mathematical expression of the equilibration operator was found that generalized some of the previously obtained results. In the present paper, we discuss this operator and itemize key points to be paid attention to in the context of the equilibration operator application. Moreover, we demonstrate that the found operator, when applied to the problem, behaves very typically for path-equilibration techniques. The main advantage of the explicit form of the operator is the high accuracy of the solution. We believe this paper can clarify the matter and give fresh insights to traffic engineers.

Keywords: Traffic flow assignment, User equilibrium, Equilibration operator.

Igor Kulachenko, Alexander Yuskov[1]

AN NPU-ACCELERATED HEURISTIC FOR THE PICKUP AND DELIVERY PROBLEM WITH TIME WINDOWS

[1] Novosibirsk State University

This study focuses on the Pickup and Delivery Problem with Time Windows (PDPTW), a challenging combinatorial problem that involves planning routes

and schedules for vehicles to pick up and deliver items within specific time limits. We present a novel method that utilizes Neural Processing Units (NPU) to accelerate the search for good-quality solutions, shifting focus from conventional CPU-based techniques. This approach takes advantage of the processing power of NPUs to enhance the problem-solving efficiency of the PDPTW. Our research shows that using NPUs can make the search process significantly faster compared to using CPUs, while still finding solutions that are very close to the best ones known so far on instances that include up to 5000 locations. In addition, our work provides a detailed look at how the NPU-powered method works, compares it with existing CPU-based algorithms, and discusses its strengths and limitations. By doing so, we contribute to the ongoing conversation about using new technologies for complex optimization problems. Our study aims to serve as a starting point for further research into using NPUs for solving such problems, highlighting both the opportunities and challenges that come with this technology.

Keywords: vehicle routing, matrix computations, parallel computations, large neighborhood search.

Suriya Kumacheva[1], **Vitalii Novgorodtcev**[2]

On one class of stochastic optimal control problems

[1] St.Petersburg State University

[2] St Petersburg State University

Among a large number of modern problems related to financial analysis and risk management, it is worth highlighting the class of problems on finding optimal portfolio control with quality criteria containing metrics such as VaR and CVaR. Focusing on the second type of metric, we note that the main difficulty in solving such problems is time inconsistency [1], which leads to the impossibility of applying dynamic programming methods directly [2]. Typically, in such cases, the original space is immersed in a space of higher dimension and the problem is reduced to the classical case [3], [4], which significantly increases the number of calculations. However, [5] presents an approach by reducing the original problem to a two-level optimization problem, where the internal problem is a classical problem, and the external problem under certain conditions imposed on the quality functional and dynamics, is solved through proximal supergradient (or gradient) methods. In this case, the quality functional contains only the terminal penalty function, which is an extremal measure [5]. In particular, such a function includes the CVaR metric, if we use the representation given in [6]. The authors also noted that it is possible to include a time penalty function in the quality functional, but only by adding it to the arguments of the terminal penalty function by representing the penalty func-

tion as a new variable. This study extends the methodology of [5] to the case of adding the quality functional of the classical formulation to the extremal measure. Such a statement can easily arise in a financial application, for example, if a manager needs to look for a control that will be penalized when the process is in certain undesirable states over time, and not just at the end. Thus, the study shows the possibility of reducing the original time inconsistent stochastic optimal control problem to a two-level optimization problem, where the external problem under certain conditions can be solved by proximal supergradient methods, and the internal problem is a classical stochastic optimal control problem.

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Keywords: stochastic optimal control, CVaR, proximal supergradient, viscosity solution, extreme measure, time inconsistency.

Viktoriya Latypova

Decision Support in Optimal Multicriteria Appointment of Reviewers in Organizing Code Review Process

The selection and appointment of the right developers for the role of code reviewers is one of the main stages of code review process. The stage is quite crucial not only for the review process itself, but for the whole software development process. The problem of appointment is much more severe in case of large IT-projects, in which a great number of change requests appears almost at

the same time. And in this case, there is a need to assign appropri-ate reviewers immediately on several change requests. And, as a result, the assignment optimization problem arises. The considerable quantity of exist-ing research is engaged in the questions concerning code review process, in-cluding code reviewer appointment for sent change request. However, in them, there is little focus on the optimal assignment, and also consideration of code reviewer's general characteristics unrelated to the specifics of the re-viewed change request. An approach to decision support in optimal mul-ticriteria appointment of code reviewers, in which code reviewer appoint-ment task is considered as multi-purpose optimization problem, the general-ized assignment problem, transformed into single-purpose one by additive folding criteria-characteristics of code reviewers, is proposed in the paper to address this gap. Together with the most commonly applied in the current re-search code reviewer's features, general expertise (as a developer and code reviewer) is also taken into account in appointing. The approach to decision support in optimal multicriteria appointment of code reviewers was success-fully tested on the generated data set.

Keywords: Code Review, Code Reviewer Appointment, Decision Support, Generalized Assignment Problem, Criteria Ranking.

Sergey Lavlinskii[1], **Artem Panin**[1], **Alexander Plyasunov, Alexander Zyryanov**

PRODUCTION AND INFRASTRUCTURE CONSTRUCTION IN A RESOURCE REGION: A COMPARATIVE ANALYSIS OF MECHANISMS FOR FORMING A CONSORTIUM OF SUBSOIL USERS

[1] Sobolev Institute of Mathematics

The paper considers the model of formation of a program for the development of the mineral resource base of a resource region based on public-private partnership. The key element of the program is the mechanism of creating a consortium of private investors, jointly implementing projects for the construction of the necessary production infrastructure. Two variants of consortium formation are considered. In the first case, the consortium is formed "from above" by the government, which creates a management company. Its goal is to find a compromise between the interests of the government and subsoil users, providing the maximum possible budgetary flow. The second variant substantially assumes the formation of the consortium "from below" on the initiative of subsoil users. In this case, private investors themselves form a management company. It ensures rational sharing of infrastructure costs and, in dialogue with the government, seeks to maximize the amount of natural resource rent they receive. Both mechanisms formally fit into the Stackelberg

game scheme. The corresponding bilevel mathematical programming problems are solved using metaheuristics based on coordinate descent. The numerical experiment carried out on real data of the Zabaikalsky Krai allows us to compare the ways of consortium formation ("from above" and "from below") and formulate practical recommendations for the economically unobvious choice of partnership architecture.

Keywords: : Stackelberg game, bilevel mathematical programming problems, strategic planning, public-private partnership, metaheuristics, coordinate descent.

Maria Lavrentyeva, Anton Ushakov[1], Igor Vasilyev[2]

AN ASSISTANT SYSTEM BASED ON LARGE LANGUAGE MODELS TO ANALYZE OPTIMIZATION MODELS THROUGH NATURAL LANGUAGE CONVERSATIONS

[1] Matrosov Institute for System Dynamics and Control Theory of SB RAS

[2] Matrosov Institute for System Dynamics and Control Theory SB RAS

Large Language Models (LLMs) have been demonstrating outstanding advances and finding new applications in diverse areas such as medicine, education, biology etc. Their ability to conduct natural language dialogues and answer arbitrary user questions is becoming a key technology for decision support systems. In this talk we present an assistant framework that allows a user to perform what-if analysis, receive explanations of certain decisions, and interpret solutions of some real-world optimization problems through natural language conversations. To avoid an expensive training process, we apply the in-context learning approach to make the LLM get familiar with the domain related to a specific optimization model. The main benefit of the framework is that it can be used by a business operator, who do not require to have any experience in operations research or mathematical programming. The framework provides answer not only for what-if scenarios but also can provide detailed statistics about solutions.

Keywords: large language models, machine learning, facility location, in-context learning.

Anna Lempert[1], Alexander Kazakov[1], Nguyen Duc Minh

A HEURISTIC ALGORITHM FOR THE PROBLEM OF GEODESIC CIRCLES PACKING

[1] Matrosov Institute for System Dynamics and Control Theory of SB RAS

The report deals with the problem of packing a given number of geodesic circles into the surface of revolution. The optimization criterion is to maximize geodesic circles' radius. This formulation is a relatively little-studied

case of the classical circle packing problem (CPP) for a simply connected set, which is relevant in connection with applications. The feature of this study is that, besides the traditional Euclidean distance between points, we also apply a specific metric that characterizes the distance of points as the time of movement between them. To solve the problem considered, we suggest a heuristic algorithm based on a spherical analogue of the Voronoi diagram and the optical-geometric approach. Illustrative numerical calculations are performed. Besides, we conduct a statistical analysis of the radii obtained from different calculations and figure out its distribution law.

Keywords: circle packing problem, geodesic circles, surface of revolution, optical-geometric approach, non-Eucledian distance.

Iuliia Leonova, Anatoly Panyukov

Statistical Study of the Accuracy of the Cycle Merging Algorithm for the Maximal Traveling Salesman Problem

The traveling salesman problem has a long history of research and is currently one of the most developed problems in the field of combinatorial optimisation. Despite the high degree of development and availability of a large number of methods, there is a need to investigate new approaches and improve existing algorithms in order to improve accuracy, speed of operation and applicability in real-world problems. This paper presents a study of the quality of a new approximate algorithm, the cycle merging algorithm, for the maximal traveling salesman problem. The results of the computational experiment for the metric maximal traveling salesman problem are given, and the accuracy of the algorithm is analysed. The regression model describing the relative error estimation for a class of metric maximal traveling salesman problem is constructed. It is shown that the constructed model is significant and satisfies the Gauss-Markov assumptions. The behaviour of the model and the predicted values allow us to speak about the asymptotic accuracy of the cycle merging algorithm for a class of metric problems.

Keywords: approximation algorithms, schemes and guarantees, computational complexity, integer programming, combinatorial optimization.

Tatiana Levanova[1], **Ivan Khmara**

Development of Tabu Search and Variable Neighborhood Search Algorithms for the robust p-median problem

[1] Sobolev Institute of Mathematics

The paper is devoted to the development of methods for solving the robust p-median problem. In the modern economy, the decision-making process must

consider the changing conditions. One way to account for such changes is to build robust models. Their purpose is to determine how much the task parameters can change so that the solution remains acceptable. The classical p-median problem is well known: it is necessary to locate p production points and serve customers in them at the lowest cost. In the robust version of the p-median problem, the stability associated with consumer demand is optimized. We consider the so-called threshold robustness. A nonlinear integer programming model is written out, and its linearization is performed. Using well-known software for this problem requires a lot of CPU time and computer RAM, so we develop approximate methods. To solve this problem, a problem-oriented versions of the Tabu Search and Variable Neighborhood Search Algorithms are proposed. Based on the ideas from the well-known library "Discrete location problems", a series of test instances were created. The parameters of the algorithms were adjusted, a comparative analysis of the quality of the developments was carried out, and the results were discussed.

Keywords: discrete location, p-median problem, robustness, local search, Tabu Search, Variable Neighborhood Search.

Yaxuan Li, Zhuo Diao[1], Zhongzheng Tang

Some Combinatorial Algorithms on the Edge Dominating Number of Hypergraphs with Minimum Degree

[1] Central University of Finance and Economics

Given a hypergraph $H = (V, E)$, a set of edges $A \subseteq E$ is an edge dominating set if every edge $e \in E \setminus A$ is adjacent to at least one edge in A. The edge dominating number $\gamma'(H)$ is the minimum cardinality of an edge dominating set in H. In this paper, we prove some upper bounds of edge dominating number for hypergraphs with minimum degree δ. (i) For $\delta \leq 4$, $\gamma'(H) \leq \frac{m}{\delta}$. (ii) For $\delta \geq 5$, $\gamma'(H) \leq \frac{m}{\delta}$ holds for hypertrees and uniform hypergraphs. (iii) For a random hypergraph model $H(n, m)$, for any positive number $\varepsilon > 0$, $\gamma'(H) \leq (1 + \varepsilon)$ holds with high probability when m is bounded by some polynomial function of n. Based on the proofs, some combinatorial algorithms on the edge dominating number of hypergraphs with minimum degree are designed.

Keywords: edge dominating number, minimum degree, upper bound.

Vasiliy Lobanov, Yulia Zakharova[1]

Evaluation of Operators in Genetic Programming for Boolean Function Approximation

[1] Sobolev Institute of Mathematics SB RAS

Boolean function approximation problems with applications in regression and classification are considered. We propose a genetic programming algorithm, where solutions (individuals) are encoded as trees. This algorithm incorporates new operators having optimized nature (construction of the initial population, crossover, mutation and selection). Operators of the algorithm are studied on various series of instances. An adaptive tuning of parameters and operators is carried out. A theoretical analysis of the computational complexity of operators and runtime analysis of the algorithm is provided. Experimental evaluation and analysis of results are also presented.

Keywords: optimized operator, functional tree, Boolean function, evolutionary algorithm.

Nadezhda Lutovinova[1], **Egor Grishin, Grigorii Gerasimov**

Scheduling Techniques for Resource-Constrained Construction Projects

[1] V.A. Trapeznikov Institute of Control Sciences of RAS

This work is devoted to the project planning, specifically for construction projects. Construction projects (especially capital development) may consist of several tens of thousands of operations. There are precedence constraints and resource constraints set. This problem is known as resource constrained project scheduling problem, which is well researched. Commercial systems such as Primavera P6 or Microsoft Project use the critical path method to minimize the duration of project execution. A literature review is presented on various formulations of this problem, as well as methods of their solution. The serial schedule generation scheme method was chosen as the most perspective method for scheduling applied large-scale projects. Comparison with the critical path method on PSPLIB examples is provided.

Keywords: RCPSP, Primavera, Critical path, Serial Schedule Generation Scheme.

Aleksandr Lyapin, Ivan Davydov[1]

Constructive heuristic for additional education school timetabling

[1] Sobolev Institute of Mathematics

The planning problem that arises in a school providing additional educational services is considered. The school offers various language courses depending on the level of knowledge of the students. The work schedule is formed on the basis of applications from potential students, which indicate free days and hours for classes. During the construction of schedule, students are grouped

together, assigned class times, rooms, and one teacher. The school's goal is twofold: to maximize the number of students attending while maintaining a high average number of students per class. We have proposed a mathematical model of the described problem in terms of integer linear programming and proposed a constructive heuristic for solving the problem. This work presents the results of comparing the proposed approach and a commercial MIP solver. The calculations were based on synthetic data based on real data.

Keywords: constructive heuristic algorithm, timetabling.

Yifen Mu, Xiaoguang Yang, Xinxiang Guo

A novel NE-solving paradigm for two-player zero-sum games

Based on our investigation of the dynamics of the Hedge-myopic system driven by the famous Hedge learning algorithm and the myopic best response by its opponent, we find that under mild conditions, the system will be cyclic after finite stages and the time-averaged strategy of the player who adopts the myopic best response within one period is just an exact NE strategy. Based on this finding, we propose a novel paradigm for solving two-player zero-sum games. Experiments show that our paradigm exhibits faster convergence, better stability and can attain precise NE convergence for many real cases.

Keywords: Repeated game, Equilibrium solving, Hedge algorithm, Myopic best response.

Tatiana Makarovskikh[1], **Mohamed Sallam**[1]

Constructing an optimal trajectory for monitoring objects

[1] South Ural State University

In our study, an algorithm for constructing the shortest route for a robot monitoring various objects from the ground is considered. This can be room monitoring, filming of industrial facilities, monitoring the condition of fruit trees, etc. During monitoring, some obstacles may occur in the robot's path: temporarily arisen and removable (people, furniture, appliances, emergency area) or permanent (walls, some huge equipment). The robot is controlled using a program developed by the authors, which stores a database of routes successfully overcome by the robot for the most accurate determination of the trajectory of the robot, taking into account obstacles encountered in its path. The developed algorithm uses artificial intelligence methods to classify obstacles.

Keywords: management tasks, monitoring, routing, trajectory, travelling salesman problem, software.

Svetlana Malakh[1], **Vladimir Servakh**[2]

THE PROBLEM OF PLANNING INVESTMENT PROJECTS WITH LENDING.

[1] Omsk Division of Sobolev Institute of Mathematics SB RAS

[2] Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, Omsk

We are working on an approach that can be used to implement large-scale projects. First, the general formulation of the resource-constrained scheduling problem (RCPSP) is described. The next option for solving the problem with the criterion of maximizing net present value (NPV) from the entire project. The only type of resource used is financial, since all other resources are exchanged in monetary terms, and cash flows can be exchanged at any time during the execution of work. Next, we consider the problem of scheduling an investment project with the possibility of attracting borrowed capital, in which the net present value of the entire project will be maximum.

The use of investment or borrowed funds makes it possible not to consider resource limitations, but you have to pay for their use. With this approach, any schedule consistent with a partial order becomes valid. The computational complexity of the problem is researched, and a pseudopolynomially solvable case is found.

Keywords: project scheduling, investment, NPV, lending.

Valeriy Marakulin[1]

DIFFERENTIAL INFORMATION ECONOMIES: REE-EQUILIBRIUM UNDER CONTRACT BASED APPROACH

[1] Sobolev Institute of Mathematics

The paper investigates economies with differential information from a contract-based approach developed in a series of author's papers. It clarifies the key notions of DIE-equilibrium and studies the most significant kinds of core and domination, and their conformity with Walrasian Expectations Equilibrium (WEE) and Rational Expectations Equilibrium (REE). We also introduce core and equilibrium with differentiated agents (well correspond to conditionally expected utilities). These notions have a close relationship with REE-equilibrium and clarify an appropriate core and a type of stability. Moreover, the paper presents new theorems on the existence of core and quasi-equilibria.

Keywords: Differentiated information, Contractual approach, WEE-Walrasian Expectations Equilibrium, REE-Rational Expectations Equilibrium.

Chiara Marciano, Mario Guarracino, Brian Daniel Bernhardt

Improved Credit Scoring model with Hyperparameter Optimization

This article investigates the application of machine learning techniques for predicting corporate default risk. In the credit scoring domain, the class imbalance problem is prevalent, with defaulted cases typically being much less numerous than non-defaulted ones. To address challenges posed by unbalanced datasets, various calibration methodologies, including data sampling strategies and Synthetic Minority Oversampling Technique (SMOTE), are explored to identify the most effective strategy for the specific case. Additionally, hyperparameter tuning plays a crucial role in enhancing machine learning mod-el performance. Therefore, we employ hyperparameter optimization techniques such as Grid Search, Random Search, Bayesian Optimization and Adaptive Particle Swarm Optimization. The primary objective is to compare these tech-niques and determine the optimal strategy for predicting company default events. Our evaluation incorporates standard parameters such as accuracy, bal-anced accuracy, specificity, sensitivity, and F1-score. Another factor to consider to discriminate the various approaches is the execution time, in order to be able to develop models that can be implemented in market products. The experimental results reveal which approaches result in significantly improved classification performance.

Keywords: Credit Scoring, Hyperparameter optimization algorithms, Oversampling technique.

Vladimir Mazalov[1], **Anna Ivashko**[2]

Optimal Stopping Strategies in Gambler's Ruin Game

[1] Karelia research center

[2] Institute of Applied Mathematical Research of the Karelian Research Centre of the Russian Academy of Sciences

We consider a game-theoretic version of the gambler's ruin problem. In each of the n steps, two players with different capitals compete over a unit of capital. The players' chances in each step are equal. Accordingly, the capital of each player can increase or decrease by one unit with equal probability. The player wins if the opponent runs out of capital. In this case, the player gets 1 as payoff. If the game has not ended within the time interval n, then the players gain nothing. At each step, the players are required to pay a value of c. Two variants of the game are examined: one where one player's capital is infinite, and the other where both players' capitals are infinite. The player's strategy is the stopping time in the game in order to maximize the expected payoff. The players' optimal stopping strategies and payoffs are determined. The numerical results of payoff simulation for different n are reported.

Keywords: random walk, Gambler's ruin problem, optimal stopping, reflection principle.

Andrey Melnikov[1], **Vladimir Beresnev**

FINDING A STATIC LEADER'S DECISION IN A COMPETITIVE LOCATION WITH A MULI-PERIOD FOLLOWER'S DECISION

[1] Sobolev Institute of Mathematics

We consider a dynamic competitive facility location problem modeling an interaction of two competing parties (Leader and Follower), who open their facilities within a planning horizon composed of several time periods. We assume that Leader opens their facilities at the beginning of the planning horizon and does not change the decision later. On the opposite, Follower can correct their decision on each of the planning horizon's time period. We propose an algorithm to compute the best Leader's solution based on a branch-and-bound scheme. Computations of upper bounds employ a high-point relaxation of the initial bi-level problem strengthened with additional constraints. Procedures to construct such constraints using auxiliary optimization problems, ensuring the strongest cuts generation, are proposed. The numerical experiments results using randomly generated instances are discussed.

Keywords: Stackelberg game, competitive location, bi-level programming, mathematical programming, dynamic location.

Dmitriy Merzlyakov[1]

IMAGE ANALYSIS OF SMALL ARCHITECTURAL FORMS USING COMPUTER VISION MODELS

[1] Novosibirsk State University

The paper considers the construction of various computer vision models for the task of detecting small architectural forms in images of playground plans, as well as methods for optimal choice between predictions of various models that would maximize average accuracy, while striving to achieve the smallest spread.

Keywords: machine learning, computer vision, convolutional neural network.

Anton Mescheryakov[1], **Oleg Khamisov**

A STUDY OF A NASH EQUILIBRIUM IN A DUAL-MARKET ECONOMIC SYSTEM

[1] Skolkovo Institute of Science and Technology

In this study, we introduce a computational model designed to find a Nash equilibrium in a dual-market economic system. The first market consists of

sellers of materials and buyers who use materials to produce goods. The second market consists of these goods sellers and buyers. The model addresses strategic trans-market interactions between sellers who aim to maximize their profits in the presence of market dynamics.

Sellers' production capacities and goods buyers' demand levels serve as constraints. Production costs are linear, while delivery costs depend on logistics. Materials and goods price curves for both markets are non-linear and serve as the main trans-market connection through which sellers can influence each other decision-making.

To achieve a Nash equilibrium, the combined dual-market optimization process allows each seller to optimize strategy in response to the other's actions while considering non-linear trans-market price curves. The iterative process continues until none of the sellers can benefit from a change in their strategy, indicating a steady state. Graphical representation of the trans-market optimization process confirms the convergence of the process towards a Nash equilibrium, characterized by stable profits.

This computational model may help expand understanding of the complex dynamics of market interactions and provide a methodological framework for analyzing strategic behavior in economic systems. We plan to expand the study by introducing non-linear production and logistics costs as well as weights and probabilities for various markets conditions and economic events to take place in order to conduct scenarios-vise forecasting of the market state.

Keywords: Nash, Equilibrium, Markets, Economics, Optimization.

Liudmila Mikhailova[1]

One Optimization Problem Induced by the Segregation Problem for the Sum of Two Quasiperiodic Sequences

[1] Sobolev Institute of Mathematics

An unexplored discrete optimization problem of summing the elements of three given numerical sequences is considered. This problem is a core, within the framework of a posteriori approach, of the noise-proof segregation problem for two independent unobservable quasiperiodic sequences, i.e., the sequences that include some non-intersecting subsequences-fragments having the predetermined characteristic properties with the limitations from below and above on the interval between two successive fragments. The segregation problem is to restore the unobservable sequences on the base of their noisy sum. In the current paper, all the fragments in a single sequence are assumed to be identical and coinciding with the given reference fragment, at that, the information about the number of fragments in it is unavailable.

It is shown constructively that, despite the exponentially-sized set of pos-

sible solutions to the optimization problem under consideration, as well as in the segregation problem, both these problems are polynomially solvable. Some numerical simulation results are given for illustration.

Keywords: Discrete optimization problem, Polynomial-time solvability, Quasi-periodic sequence, Segregation, Detection, One-microphone signal separation.

Ilya Minarchenko[1]

О поиске равновесия по Нэшу в немонотонных квадратичных играх

[1] Melentiev Energy Systems Institute SB RAS

Рассматривается немонотонная некооперативная игра в нормальной форме с квадратичными функциями выигрыша и множествами стратегий, заданными выпуклыми многогранниками. С помощью функции Никайдо-Исоды задача на поиск равновесия сводится к задаче минимизации неявно заданной невыпуклой функции. Для поиска глобального оптимума предлагается метод, основанный на аппроксимации целевой функции нелинейными опорными минорантами. В случае, если функции выигрыша строго вогнуты по собственным переменным, дополнительно может быть рассмотрен метод локального поиска. Заметим, если игра невогнута, то существование равновесия не гарантируется. Однако предлагаемая методика позволяет в ходе вычислительной процедуры установить отсутствие равновесий, если игра не имеет таковых.

Keywords: Nash equilibrium problem, nonconvex game, nonmonotone game, global optimization.

Sofia Mirkina

Quasi-regular covers of graphs with bounded treewidth

For a graph $G = (V, E)$, an integer k, and a set of integers D, a D-regular k-cover is a set of edges $S \subseteq E$ such that G' = (V, S) consists of at most k connected components and each vertex in G' has a degree belonging to D. For example, in case when $k = 1$ and $D = \{2\}$ we need to find the set of edges such that the subgraph is the Hamiltonian cycle. We show an algorithm for finding D-regular k-covers in $2^{(O(tw*log(d_m))} * poly(k, n)$ time, where $n = |V|$, tw is the treewidth of G and d_m is the biggest integer in D.

Keywords: fixed-parameter algorithm, NP-hard problem, parameterized complexity, graph factors, Hamiltonian cycle.

Aleksandr Morshinin[1]

New integer linear programming models for a variant of correlation clustering problem

[1] Sobolev Institute of Mathematics SB RAS

In correlation clustering (cluster editing) we must split vertices of a graph into clusters based on their similarity, which is given by the edge structure of the graph. There are different formulations of the problem: with a constraint on the number of clusters, their cardinality, etc. The problems under consideration are NP-hard. Thus we need to build exact algorithms and mathematical programming models. We consider a problem in which the number of clusters does not exceed a predefined constant. New integer linear programming models are constructed for this problem. We also provide an analysis of the experimental study, which aims to compare new models with known models.

The research was supported by Russian Science Foundation grant N 22-71-10015.

Keywords: correlation clustering, integer linear programming, cluster graph.

Zhinlin Mu[1], **Tiande Guo, Xin Sun**[1]

A Fast Algorithm for Submodular Maximization with a Matroid Constraint

[1] University of Chinese Academy of Sciences

We present a fast algorithm for maximizing monotone submodular function over a matroid constraint. It is a $(1 - 1/e - \epsilon)$ approximation algorithm with nearly-linear time complexity, where $\epsilon > 0$. We improve the complexity of the LazySamplingGreedy algorithm proposed by Ene and Nguyen, which is the first nearly-linear time algorithm with the same guarantee above. It made $O(n \ln n \ln(r/\epsilon)/\epsilon)$ calls to the value oracle so that the approximation guarantee above can be obtained with a probability of at least $2/3$, where n is the cardinality of the given ground set and r is the rank of the matroid constraint. We improve the complexity and propose the FastLazySamplingGreedy algorithm, which can produce the same approximation ratio with a probability of at least $3/4$ and only made $O((r \ln n + n) \ln(r/\epsilon)/\epsilon)$ calls to the value oracle.

Keywords: Monotone submodular maximization, Matroid constraint, Complexity, Approximation algorithm.

Elena Musatova[1], **Alexander Lazarev**[2]

A single machine problem of minimization of external resources costs

[1] Trapeznikov Institute of Control Sciences

[2] Institute of Control Sciences

We consider the following problem. There is a set of jobs that must be serviced on a single machine. A graph of precedence relations between jobs that sets a partial order of jobs is given. There are also several subsets of the jobs. Each subset requires the use of its own additional resource. This resource is leased from the start of the first job from the subset and returned after the last job of the subset is completed. It is necessary to construct a schedule for which the cost of external resources is minimal. This problem arises in many areas of production, where equipment rentals and subcontracts are used. We have proved the NP-hardness of the problem, and propose an algorithm for solving this problem, the complexity of which depends polynomially on the number of jobs, but exponentially on the number of external resources. Computational experiments show high efficiency of the algorithm in the case of a small number of resources or in the case of a high-density graph of precedence relations.

Keywords: scheduling theory, single machine problem, NP-hard problems, downtime of resources minimization.

Stepan Nazarenko[1]

UPDATED ESTIMATES FOR SEVEREAL ALGORITHMS FOR PACKING 2-BAR CHARTS IN A STRIP

[1] Novosibirsk State University

We consider a Two-Bar Charts Packing Problem in which it is necessary to pack bar charts consisting of two bars in a unit-height strip of minimum length. Each bar has a height of at most 1 and unit length. The problem under consideration is NP-hard. The problem is a generalization of the Bin Packing Problem and Two-Dimensional Vector Packing Problem. This paper proves updated accuracy estimates and time complexity for several previously developed polynomial approximation algorithms for the Two-Bar Charts Packing Problem and particular cases of the problem. We show the attainability of the obtained estimates. Furthermore, we consider a problem of packing an unlimited number of bar charts belonging to k different types. We propose a polynomial algorithm to solve the problem in case $k = const$.

Keywords: bar chart, strip packing, a priori estimates, attainability.

Andrei Nikolaev[1]

ON 1-SKELETON OF THE CUT POLYTOPES

[1] P.G. Demidov Yaroslavl State University

Given an undirected graph $G = (V, E)$, the cut polytope $\mathrm{CUT}(G)$ is defined as the convex hull of the incidence vectors of all cuts in G. The 1-skeleton of $\mathrm{CUT}(G)$ is a graph whose vertex set is the vertex set of the polytope, and the

edge set is the set of geometric edges or one-dimensional faces of the polytope.

We study the diameter and the clique number of 1-skeleton of cut polytopes for several classes of graphs. These characteristics are of interest since they estimate the computational complexity of the max-cut problem for certain computational models and classes of algorithms.

It is established that while the diameter of the 1-skeleton of a cut polytope does not exceed $|V| - 1$ for any connected graph, the clique number varies significantly depending on the class of graphs. For trees, cacti, and almost trees (2), the clique number is linear in the dimension, whereas for complete bipartite and k-partite graphs, it is superpolynomial.

Keywords: Max-cut problem, Cut polytope, 1-skeleton, Diameter, Clique number, Connected component, Chromatic number.

Natalia Obrosova[1], **Alexander Shananin, Alexander Spiridonov**

LABOR DEMAND ANALYSIS IN PRODUCTION NETWORKS WITH SUBSTITUTION OF INPUTS

[1] Federal Research Center Computer Science and Control of the Russian Academy of Sciences

Modern processes of deglobalization and import substitution lead to the need for restructuring the sectoral structure of local economies. In conditions of limited resources, such changes can lead to imbalances in demand and supply, thereby increasing inflation risks. Traditionally, models of interindustry balance are used to analyze such problems in various scenarios of macroeconomic development. The technology of Input-Output Analysis, developed in the second half of the last century and based on the linear Leontief model, requires modification in modern conditions to account for the substitution of production factors. This work proposes a mathematical model of interindustry balance, based on solving variational inequalities and Young dual problems, and determining the competitive equilibrium in the space of goods and prices under given scenario conditions. One of the significant risks in conditions of structural changes is the disbalance of demand and supply in the labor market. We present the application of the developed nonlinear interindustry balance model to labor demand evaluation in sectors of the Russian economy under changing scenario conditions.

Keywords: input-output model, variational inequality, competitive equilibrium, Young duality, convex optimization.

Omoehin Joseph Olorunju, Oluwafemi Oluwaseyi James, Ajayi Omolola

ON HYBRID FORMULATION OF PERTURBATION PARAMETERS IN GRADIENT METHOD

The work considered the hybrid formulation of the perturbation term (λ) in Gradient Methods using the theoretical formulated convergence rate of the Conventional Conjugate Gradient Method (CGM). The work considered nonlinear quadratic functional the result obtained is far better than the existing ones.

Keywords: perturbation parameters, gradient method, nonlinear quadratic functional.

Andrei Orlov[1]

Triangles of Conflicts and Hexamatrix Games

[1] Matrosov Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences

One of the tools for modeling and studying real-life conflicts is the mathematical apparatus of Game Theory. In this case, the numerical search for equilibrium strategies of players (conflicting sides) is an urgent problem of modern mathematical optimization. The work addresses one class of finite games (with finite numbers of strategies for each player) – Polymatrix Games of E.B. Yanovskaya. Namely, we study in detail the 3-players Polymatrix Games, the so-called Hexamatrix Games (HG). Several examples of 3-sides real-life conflicts are presented and formulated as HG.

To find a Nash Equilibrium in formulated games, we use the optimization approach, when the problem is transformed into a nonconvex optimization problem with bilinear and d.c. structure. The latter is solved by the A.S. Strekalovsky's Global Search Theory (GST) for (d.c.) optimization problems with goal functions represented as the difference of two convex functions.

The research was supported by RSF (project No 24-41-03004).

Keywords: N-sides conflicts, Polymatrix games, 3-sides conflicts, Hexamatrix games, Nash equilibrium, Nonconvex optimization approach to hexamatrix games, Global Search Theory.

Andrei Orlov[1]

On a Global Search in Bilevel Optimization Problems with a Bimatrix Game at the Lower Level

[1] Matrosov Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences

The work addresses one class of bilevel optimization problems (BOPs) in an optimistic statement with an equilibrium at the lower level. Namely, we study BOPs with a convex quadratic objective function under linear constraints at the upper level and with a parametric bimatrix game at the lower one, where we need to find a Nash equilibrium. First, we transform the original bilevel problem

into a single-level nonconvex optimization problem by replacing the lower level with its optimality conditions. Then we apply the Exact Penalization Theory and Global Search Theory (GST) for general d.c. optimization problems to the resulting problem.

According to the standard research of nonconvex problems by the GST, we construct the d.c. representations of all nonconvex functions from the original statement, formulate the Global Optimality Conditions in terms of reduced penalized problem, and develop local and global search method taking into account the specific of problem in question. The main feature of the developed methods consists in the possibility of varying the penalty parameter within the methods themselves.

Keywords: Bilevel optimization, Bimatrix game, Nash equilibrium, Global Search Theory, Global Search Scheme, Local Search, Bilevel problems with an equilibrium at the lower level, Optimistic solution, Exact Penalization Theory.

Anna Panasenko[1]

Approximation Scheme for a Sequence Weighted 2-Clustering with a Fixed Center of One Cluster

[1] Sobolev Institute of Mathematics

We consider a problem of 2-clustering of a finite sequence of points in Euclidean space. In this problem, we need to partition an input sequence into two subsequences (clusters) minimizing weighted intracluster sums of the squared distances from clusters elements to their centers. The center of the first cluster is its centroid, while the center of the second one is the origin. Some constants are used to define the boundaries of the difference between any two subsequent indices of the elements in the cluster with an unknown centroid. The problem is a generalization of previously considered problems with specific weights, as it includes the input for the weight factors for both intracluster sums. In the paper, we present an approximation algorithm. It is based on an adaptive-grid-approach and dynamic programming. If the dimension of the space is fixed, this algorithm constitutes an FPTAS.

Keywords: Euclidean space, Weighted 2-clustering, Quadratic variation, NP-hardness, Approximation algorithm, FPTAS, Sequence clustering problem.

Artem Panin[1], **Evgeniy Brazhnikov**

Pattern-based greedy algorithms for a temporal bin packing problem with placement groups

[1] Sobolev Institute of Mathematics

We consider a temporal bin-packing problem where bins are servers using the Non-Uniform Memory Access architecture and items are virtual machines. Each user can create one or more placement groups consisting of subgroups called partitions and assign the created virtual machines to them. Partitions of the same group are in conflict with each other. Virtual machines from different partitions of the same group must be placed in different failure domains. This can help reduce the likelihood of correlated failures, improving your application performance and availability. In the problem formulation under study, the failure domain is a rack of servers.

We propose several algorithms to solve the problem using a pattern-based approach and greedy heuristics. The algorithms differ in their approaches to constructing patterns, selecting an initial time moment, solving a single moment problem, and extending the solution to the right and left.

Keywords: Temporal Bin Packing, Virtual machines, Placement groups, Patter-based approach, Greedy algorithm.

Artem Panin[1], **Pavel Borisovsky**[2], **Anton Eremeev**[3], **Maksim Sakhno**

Temporal Bin Packing Problems with Placement Constraints: MIP-Models and Complexity

[1] Sobolev Institute of Mathematics

[2] Sobolev Institute of Mathematics SB RAS

[3] Dostoevsky Omsk State University

In this paper, we investigate new problem statements, generalizing the Temporal Bin Packing Problem (TBPP) with possible applications in cloud computing. We suppose that items are organized into *batches*. All items in the same batch are placed simultaneously. In cloud computing, items correspond to virtual machines (VMs) and batches correspond to user requests for VM placement. In addition, cloud users can create *placement groups* consisting of VMs united by a single placement constraint named *cluster*: at any moment in time, VMs from the same placement group must be hosted on the same rack of servers. In this paper, we consider servers as one-dimensional bins.

We investigate the computational complexity and inapproximability of different formulations of the TBPP with cluster placement constraint and suggest mixed integer programming models for them.

Keywords: Temporal Bin Packing, Placement groups, Virtual machines, Complexity, Inapproximability, MIP model.

Panos Pardalos[1,2]

AI and Optimization for a Sustainable Future

[1] University of Florida, Gainesville, USA
[2] HSE, Nizhny Novgorod, Russia
Advances in AI tools are progressing rapidly and demonstrating the potential to transform our lives. The spectacular AI tools rely in part on their sophisticated mathematical underpinnings (e.g. optimization techniques and operations research tools), even though this crucial aspect is often downplayed. In this lecture, we will discuss progress from our perspective in the field of AI and its applications in Energy systems and Sustainability.

Keywords: energy system, sustainability, artificial intelligence.

Leon Petrosyan[1], Yaroslavna Pankratova[2]

DIFFERENTIAL NETWORK GAMES WITH DIFFERENT TYPES OF PLAYERS BEHAVIOR

[1] Saint Petersburg State University
[2] St Petersburg State University

In the paper, a cooperative differential network game with infinite duration in which players follow different types of behavior (to cooperate or to act individually in their own interests) is considered. As solutions the core and the Shapley value are proposed, and existence of the core is proved. The results are illustrated on an example.

Keywords: the Shapley value, Dynamic network game, core.

Darya Piskeeva, Alexander Plyasunov, Artem Panin[1]

VND METAHEURISTICS FOR THRESHOLD STABILITY IN A FACILITY LOCATION AND DISCRIMINATORY PRICING PROBLEM

[1] Sobolev Institute of Mathematics

Threshold stability of a facility location and discriminatory pricing problem is studied. First, the company locates facilities and assigns service prices to clients. Each client selects the facility with the lowest cost of purchasing and transporting products, and makes a purchase if the cost does not exceed its budget. The goal is to find such a feasible solution and such a maximum deviation from the clients' budgets that the company's profit is not less than a given threshold. Three approximate algorithms based on Variable Neighborhood Descent (VND) are developed. Facility locations are iteratively enumerated. For each location, the pricing problem is solved. The algorithms differ in the criterion for comparing different facility locations: 1) maximizing the company profit; 2) maximizing the threshold stability radius; and 3) a combination of 1) and 2). The work was supported by the Russian Science Foundation (project 23-21-00424).

Keywords: threshold stability, VND, facility location, pricing.

Leonid Popov[1]

How to use barriers and symmetric regularization of Lagrange function in analysis of improper nonlinear programming problems

[1] Krasovskii Institute of Mathematics and Mechanics UB RAS

For numerical analysis of improper mathematical programming problems, a new approach is presented, based on ideas of symmetrical regularization of their Lagrange functions with additional barrier terms for both groups of variables, primal and dual. It makes it possibly not to specify in advance the type of incorrectness of the problem being solved as well as apply second-order optimization methods for them. The description of the approach, convergence theorems and meaningful interpretation of the obtained generalized solutions are given.

Keywords: nonlinear programming, improper (ill-posed) problems, penalty and barrier methods, regularization.

Artem Pyatkin[1]

Assimptotically optimal construction of graphs with maximum number of open triangles and small number of edges

[1] Sobolev Institute of Mathematics

An open triangle (OT) is an induced subgraph on 3 vertices having exactly 2 edges. It is the same as an induced 3-path. It is known that the maximum number of OT's in a n-vertex graph can be found in a complete bipartite graph with equal or almost equal parts (depending on the oddity of n). However, if the number of edges m is also fixed then the problem of finding graphs with the maximum number of OT's is still open in case of $m > n$. In this paper we prove that in the class of graphs with $m = n + c$, where c is a constant and n is large enough, there is the only graph with the maximum number of OT's.

The research is supported by the state contract of the Sobolev Institute of Mathematics (project FWNF-2022-0019).

Keywords: graph, open triangle, construction.

Varvara Rasskazova[1]

Sequential constraints scheme in distribution planning problem

[1] Moscow Aviation Institute (National Research University) (MAI)

The paper is devoted to investigation of the priority-constrained distribution planning problem. A set of resources needs to be allocated to the set of tasks specified volume parameters. In the priority-constrained formulation, each resource becomes available in a strictly predetermined order. In particular, a resource

cannot be utilized until all resources with higher priority have been exhausted. To solve this problem, a basic model of integer linear programming (ILP) is developed, with each variable corresponding to a potential assignment of a resource to a task. At this stage, a related problem arises, which involves finding the maximum number of tasks that can be fulfilled given a fixed number of resources and predetermined priorities. To tackle this, an auxiliary ILP problem is consideed. It is found that the solution to the auxiliary problem determines the desired number of tasks, which forms the input data for the construction of the basic model. Hence, solving the original planning problem involves sequentially solving the auxiliary ILP problems and the basic problem at each iteration until all available resources are exhausted (or all tasks are fulfilled). This proposed approach fully captures the specifics of sequential priority-constrained case and significantly reduces the dimensionality of the basic ILP model at each iteration. The developed scheme has been implemented in Python, using the open-source PuLP library and the CBC solver to solve the arising ILP problems. Computational experiments were conducted within the context of solving real-world priority-constrained distribution planning problems that arise in the metallurgical industry.

Keywords: resource planning, priority-constrained case, integer linear programming model, scheme of sequential constraints.

Alexey Ratushnyi[1]

A Reconstruction Algorithm for a Temporal Bin Packing Problem with Placement Groups

[1] Sobolev Institute of Mathematics

We consider an NP-hard temporal bin packing problem with a number of items, or so-called virtual machines. For each machine, we have information about the creation time, the deletion time, the number of cores, and the amount of RAM. All items are divided into two types: large and small. Each container (server) consists of several different parts called NUMA nodes. Small items can be completely placed on one of the nodes, whereas large items are divided into two identical halves, which are placed on different nodes. Servers are located in racks, and some virtual machines are grouped together. Each group is divided into partitions. Virtual machines from different partitions of the same group conflict with each other, which means that they cannot be placed on the same rack. This approach is necessary for the fault tolerance of the system. We want to efficiently pack all virtual machines into the minimum number of racks. For this problem, we design a two-step heuristic. At the first step, we build a simple solution without any conflict constraints. Then, using local search, we iteratively get rid of constraint violations for each group. We carry out computational experiments

on many different instances with up to 35000 virtual machines.

Keywords: bin packing problem, virtual machines, conflicts, placement groups, cloud computing.

Enkhbat Rentsen[1]

RECENT ADVANCES IN MINIMAX PROBLEM AND BIMATRIX GAMES

[1] The Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences

In this paper, we first survey recent developments in game theory, and then focus on minimax problems and bimatrix games. New optimality conditions for maxmin and minimax problems have been considered. Recently introduced new equilibriums such as Anti-Berge and Anti-Nash equilibriums are discussed with connection to optimization problems. We examine bimatrix games from a view point of global optimization. Recent results in multi player polymatrix game have been given. The existence theorems of all equilibrium are provided. Also, a triple matrix game is introduced and existence of Nash equilibrium in the game has been given.

Keywords: optimization, game theory, optimality condition, maxmin problem, minimax problem, equilibriums, bimatrix game, payoff function, strategy.

Anna Rettieva[1]

COOPERATION MAINTENANCE IN DYNAMIC MULTICRITERIA GAMES WITH APPLICATION TO TRANSPORT NETWORKS

[1] Institute of Applied Mathematical Research of the Karelian Research Center of RAS

We consider a dynamic, discrete-time, game model where the players seek to optimize different criteria. To construct a multicriteria Nash equilibrium the bargaining solution is adopted. To obtain a multicriteria cooperative equilibrium, a modified bargaining scheme that guarantees the fulfillment of individual rationality conditions is applied. Two approaches to maintain cooperative behavior, namely, the cooperative incentive equilibrium and the time-consistent payoff distribution procedure, are considered. A dynamic bi-criteria transportation problem is investigated to illustrate the solution concepts proposed.

This research was supported by the Russian Science Foundation: grant No. 22-11-20015, https://rscf.ru/project/22-11-20015/, jointly with support of the authorities of the Republic of Karelia with funding from the Venture Investment Foundation of the Republic of Karelia.

Keywords: dynamic games, multicriteria games, resource management problem, time-consistency, incentive equilibrium.

Roman Rudakov[1], **Yuri Ogorodnikov**[2], **Michael Khachay**[2]

Branching algorithms for the Reliable Production Process Design Problem

[1] N.N. Krasovskii Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences (IMM UB RAS)

[2] Krasovsky Institute of Mathematics and Mechanics

In the well-known Subgraph Homeomorphism Problem (SHP), it is required to make a homeomorphic embedding of some pattern digraph P into the given target digraph G. Such an embedding is performed by some one-to-one map f defined on the set V(P) such that, for each arc (v,u) of P, there exists an elementary f(v)-f(u)-path in G, and all such paths are vertex-disjoint. In this paper we consider the proposed recently Reliable Production Process Design Problem (RPPDP), which generalizes the SHP in the following way: (i) graph P is supposed to be acyclic, while G is edge- and node-weighted, which assigns costs to the considered embeddings; (ii) the mutual uncrossing constraint for f(v)-f(u)-paths is slightly relaxed; (iii) for a given number k, we are requested to find k vertex-disjoint homeomorphic images of the pattern P, such that the largest cost of the obtained images has the minimum value. The RPPDP has applications in manufacture management planning, where the decision maker is aimed to propose a family of plans tolerant to possible faults in production units and supply chains. We propose the first branch-and-bound and branch-and-price algorithms for the RPPDP. The results of numerical experiments demonstrate high performance and mutual complementarity of the proposed algorithms.

Keywords: Subgraph Homeomorphism Problem, reliable supply chains, branch-and-price.

Maria Sakhno[1]

Investigation of operators and parameters in evolutionary algorithms for one scheduling problem with resource constraints

[1] Sobolev Institute of Mathematics SB RAS

We propose an evolutionary algorithm for a scheduling problem on permutations with resource constraints. It has various tuning and adaptive parameters. Several variants of selection, mutation and crossover operators are implemented for the algorithm. We test different schemes of the algorithm and evaluate the impact of these operators and parameters on the results for instances with different structure. The computational complexity of the operators is also investigated.

The research was supported by Russian Science Foundation grant N 22-71-10015, https://rscf.ru/en/project/22-71-10015/.

Keywords: scheduling, evolutionary computational, parameter, adaptive technique.

Ivan Samoylenko

THE MODEL OF TWO-LEVEL INTERGROUP COMPETITON

At the middle of the 2000-th, scientists studying the functioning of insect communities identified four basic patterns of the organizational structure of such communities: (i) Cooperation is more developed in groups with strong kinship. (ii) Cooperation in species with large colony sizes is often more developed than in species with small colony sizes. And small-sized colonies often exhibit greater internal reproductive conflict and less morphological and behavioral specialization (iii) Within a single species, brood size (i.e., in a sense, efficiency) per capita usually decreases as colony size increases. (iv). Advanced cooperation tends to occur when resources are limited and intergroup competition is fierce. Thinking of the functioning of a group of organisms as a two-level competitive market in which individuals face the problem of allocating their energy between investment in intergroup competition and investment in intragroup competition, i.e., an internal struggle for the share of resources obtained through intergroup competition, we can compare such a biological situation with the economic phenomenon of "coopetition" — the cooperation of competing agents with the goal of later competitively dividing the resources won in consequence In the framework of economic researches the effects similar to (ii) — in the framework of large and small group competition the optimal strategy of large group would be complete squeezing out of the second group and monopolization of the market (i.e. large groups tend to act cooperatively) and (iii) - there are conditions, in which the size of the group has a negative impact on productivity of each of its individuals (this effect is called the paradox of group size or Ringelman effect). The general idea of modeling such effects is the idea of proportionality - each individual (an individual/rational agent) decides what share of his forces to invest in intergroup competition and what share to invest in intragroup competition. The group's gain must be proportional to its total investment in competition, while the individual's gain is proportional to its contribution to intra-group competition. Despite the prevalence of empirical observations, no game-theoretic model has yet been introduced in which the empirically observed effects can be confirmed. This paper proposes a model that eliminates the problems of previously existing ones and the simulation of Nash equilibrium states within the proposed model allows the above effects to be observed in numerical experiments.

Keywords: intergroup competiton, Nash equilibrium state, experiment.

Oleg Savchuk, Fedor Stonyakin[1], Sergei Puchinin, Mohammad Alkousa[2], Alexander Gasnikov

Numerical Methods for Variational Inequalities and Saddle Point Problems with Relative Inexact Information

[1] Moscow Institute of Physics and Technology

[2] Moscow Institute of Physics and Technology

In this work, we consider the variational inequality problem with Lipschitz-continuous and strongly monotone operators and the access available only to relative inexact information of these operators. To solve such a problem, we consider projection and extra-gradient methods with relative error values of the operator at the current points in each iteration. Theoretical estimates for the quality of the solution (the last iterated point) were obtained in the case when the problem is constrained. When the problem is unconstrained (i.e., the feasible set coincides with the whole space) we proved the linear convergence rate to both methods under consideration with tight conditions on the relative error parameter for which we conserve the linear convergence rate. We also consider saddle point problems with the two-sided Polyak–Lojasiewicz condition and propose a gradient descent-ascent type method for them which converges at a linear rate. Some numerical experiments, for both variational inequalities and saddle point problems, were conducted to demonstrate the effectiveness of the proposed algorithms and to confirm the theoretical results in the paper.

Keywords: Variational Inequality, Projection Method, Extra-Gradient Method, Strongly Monotone Operator, Saddle Point Problem, Polyak–Lojasiewicz condition, Relative Inexactness.

Vladimir Servakh[1]

The polynomially solvable case of single machine problem with preemption

[1] Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, Omsk

This is an analysis of the problem of minimizing the total weighted completion time for jobs of the same processing time on a single machine, given the arrival times of the jobs and the possibility of preemption. At present, the computational complexity of this problem is unknown.

Keywords: scheduling theory, single machine, release dates, preemption, identical processing times.

Sergey Sevastyanov[1]

On the uselessness of Supercomputers for solving discrete optimization problems by means of some "theoretically efficient" algorithms

[1] Sobolev Institute of Mathematics SB RAS

Which of well-known "theoretically efficient" algorithms (with guaranteed accuracy bounds and "polynomial" running time) designed for well-known discrete optimization problems are really efficient and suitable for solving practical examples of those problems? What should be a test for checking the real efficiency of an algorithm? Should it be performed on a modern Supercomputer? On a global (world-wide) System of Supercomputers? On a Universe-wide System of such Supercomputers? We will try to find answers to these questions by testing (and comparing) two well-known theoretical approximation algorithms designed for solving one classical problem of scheduling theory - the Job Shop problem to the minimum of schedule length. We took for testing one of the two algorithms (the "most advanced" one, providing a $(2+e)$-approximation for any $e > 0$) and quite a small problem instance with 10 machines and 10 operations per job. We have also made several agreements, as to: A. That our small instance is being solved in a System, consisting of parallel SUPERCOMPUTERS ("SC", for short), each SC having the maximal (known for today) capacity of $2 \cdot 10^{18}$ flops. B. That there are 10 billion such SUPERCOMPUTERS in our System on the Globe, i.e., about one "personal Supercomputer" for every inhabitant of the Earth. (By the way, ONE such modern SC occupies an area of $> 1000m^2$, which results in total $> 10.000.000km^2$.) C. That in each of 10 trillion galaxies (in the visible part of the Universe) there are 100 billion stars, and each star has 10 planets, on each of which we have placed an SC-system similar to that of the Earth. Totally, 10^{25} Globe SC-Systems, forming the Computer System of the Universe (CSU, for short). D. That we have the opportunity to parallelize our problem, equally dividing the load between all computers of the CSU, and that each SC can work as long and continuously as desired. E. That we measure the working time of our CSU in time units "AU" (Age of the Universe). I took the value of this amount to be 15 billion years. Finally, we decided not to bother our System with finding a $(2+e)$-approximate solution for an arbitrarily small e, but took $e = 100$ (allowing the System to find a solution 100 times worse than the optimal one). What do you think, how long it will take our CSU to find such a solution? The results of our estimation may surprise experienced discrete optimization people and make them think about questions like: "What the Supercomputers are needed for?" (or "How and where should they be used, in fact?"), "Which "theoretically efficient" methods should be treated as really effective?", and "How should they be tested for actual effectiveness and practicality?"

Keywords: discrete optimization problem, approximation, polynomial-time algorithm, supercomputer, scheduling, job shop.

Ruslan Sevostyanov[1]

Filtering Correction for Robotic Arms Multipurpose Regulators

[1] Saint-Petersburg State University

The paper is devoted to the problem of compensating the external disturbances while stabilizing the robotic arm in the specified position by controlling motor torques directly. Disturbance in form of the polyharmonic function is of particular interest here. For example, it can represent the hull vibrations or the motion oscillations in case when arm is mounted on the moving platform. The goal is to minimize the control reaction to such disturbance while maintaining the same stabilization properties in order to save the resource of the robot motors. Stabilization of the robot is achieved through the usage of the special multipurpose control structure. It is most useful in case when there is a set of requirements specified for the motion of the plant in different operating modes. The main advantage of such structure is that it can be synthesized step by step moving from one mode to another, so the initial complicated problem is divided into several simpler tasks. Robotic arms have nonlinear mathematical models so in order to use multi-purpose regulator here feedback linearization is applied first. The main result of the paper is the synthesis method of the dynamic corrector which is important part of the multipurpose regulator. The efficiency and the corresponding prob-lems of the proposed method are demonstrated by computer model experiments.

Keywords: Feedback Linearization, Multipurpose Regulator, External Disturbances, Robotic Arms.

Syed Zamin Shah, Ali Muhammad, Nusrat Ayesha

Advancing Machine Learning Performance: A Comprehensive Investigation of Optimization Strategies for Enhanced Accuracy, Computational Efficiency, and Robustness

The current research investigates recent progresses in optimization algorithms and methods to improve the performance of machine learning models and data analysis procedures. Increasing computational demands and complex datasets are challenging the traditional methods to address these matters effectively. This research work explores the use of accuracy of machine learning algorithms and data analysis techniques and advanced optimization strategies to potentially improve the efficiency. Therefore, this research employs a comprehensive approach that includes an extensive literature review, strict selection criteria for optimization algorithms, various experimental setups, and a number of performance metrics. The results show significant improvement in the accuracy, computational efficiency, and robustness of the models. The Metaheuristic Ensemble Optimization (MEO) strategy is shown to be a successful approach to significantly improve all the models' overall performance, and the Adaptive Learning Rate (ALR) strategy

is shown to reduce training time by an order of magnitude. The findings are interpreted and discussed along with their implications, limitations, and possible real-world applicability. This research contributes to the current state-of-the-art in the field by providing deep insights into the latest optimization strategies, and provides guidance to researchers and physicians to optimally select a strategy to improve the performance of their machine learning models and data analysis processes.

Keywords: Metaheuristic Ensemble Optimization (MEO), Machine Learning, Big Data, Adaptive Learning Rate(ALR), Optimization Strategies..

Sofia Shperling, Yury Kochetov[1]

RANDOMIZED GREEDY STRATEGY WITH CORNER FILLING FOR THE IRREGULAR 2D BIN PACKING PROBLEM

[1] Sobolev Institute of Mathematics

We are given a finite set of irregular items and an unlimited number of identical rectangular bins. Items presented as two-dimensional, possibly non-convex polygons. The length and width of bins are given. We need to pack all items without overlapping and minimize the number of used bins. The rotations of items by 0, 90, 180, and 270 angles are allowed. In the packing algorithm, we apply a fitness function based on the gravity center of the packing area and idea of the well-known sky-line approach for regular packing problem. To improve the result, we apply a special rule to fill the corners of bins. This rule is based on a function to evaluate how good a corner of each item fits in the corner of the bin. We select a subset of items with the best values of this function and randomly choose one of them to pack in the corner. We present computational results with filling of one, two, three, and four corners of some bins before packing. The comparison with well-known algorithms is presented on the real test instances from Novosibirsk car mats manufacturer company with up to 50 items.

Keywords: 2D bin packing problem, irregular item, fitness function based algorithm, local search.

Roman Shtykov

PROCESSES MODELING BY THE BOUNDARIES OF THE ANTI-SURGE ZONE TO OPTIMIZE THE CONTROL OF THE OPERATING MODES OF THE GAS COMPRESSOR STATION

Simulation of prevention of surging processes at compressor stations. In the framework of the boundary layer theory, to obtain an analytical solution to the problem of the boundary layer of compressible gas between the slightly curved blades of the impeller. To determine the distribution of the gas flow

velocity between the boundary layers of two adjacent impeller blades, ensuring a continuous flow of gas in the boundary layers, which prevents the appearance of surging due to sudden changes in the flow.

Keywords: modeling, compressor station, calculation, pressure, surge, automated control, regulation.

Elena Sofronova

Optimal Control Problem of Traffic Flows by Different Types of Signal Timing

The optimal control problem of traffic flows in an urban road network is solved. The control object is the traffic flow passing through the urban network. The road network is presented as a directed graph whose nodes correspond to road sections and whose edges correspond to manoeuvres at intersections. A universal recurrent traffic flow model, based on the controlled networks theory, is applied. The traffic flow is controlled by the duration of the working phases of the traffic lights. The considered types of signal timing are: within a fixed cycle length of traffic lights, within a multicycle, within an arbitrary cycle length. The state of an object is a numerical estimation of the flow on each section of the road at each control step. The solution of the problem is the Pareto front of optimal solutions according to several criteria. An example of the solution of the multi-objective optimal control problem for field data of real intersections in Moscow is given.

Keywords: Traffic flow control, Optimal Control, Traffic flow model, Multi-objective optimization.

Vladimir Solodkin[1], Savelii Chezhegov, Ruslan Nazykov, Aleksandr Beznosikov[2], Alexander Gasnikov

Accelerated Stochastic Gradient Method with Applications to Consensus Problem in Markov-Varying Networks

[1] MIPT

[2] Moscow Institute of Physics and Technology

Stochastic optimization is a vital field in the realm of mathematical optimization, finding applications in diverse domains ranging from operations research to machine learning. In this paper, we introduce a novel first-order optimization algorithm designed for scenarios where Markovian noise is present, incorporating Nesterov acceleration for enhanced efficiency. The convergence analysis is performed by using an assumption on noise depending on the distance to the solution. We also delve into the consensus problem over Markov-varying networks, exploring how this algorithm can be applied to achieve agreement among multiple agents with differing objectives during the changes into communication system. To show the

performance of our method on the problem above, we conduct experiments to demonstrate the superiority over the classic approach.

Keywords: convex optimization, stochastic optimization, Markovian noise, accelerated methods, decentralized communications.

Pavel Sorokovikov, Aleksander Gornov, Tatiana Zarodnyuk

Numerical Investigation of the Swarm Intelligence Algorithm Obtained Using ChatGPT for Univariate Global Optimization

This paper explores the possibility of designing an efficient global optimization algorithm using an artificial intelligence chatbot, ChatGPT. The main idea is to use the swarm intelligence metaheuristic method, which operates on a set of particles in a swarm, to solve non-local univariate search problems. Testing was carried out on a set of one-dimensional non-convex functions with varying numbers of local optima. The results demonstrate that this algorithm can be used to solve multi-extremal optimization problems. The paper presents the results of a numerical study on the properties of this algorithm in comparison with the Piyavsky method, the Strongin method, and a combination of the Strongin with the Parabolas methods. The performed computational experiments testify to the suitability of the obtained swarm intelligence algorithm for solving univariate non-local search problems. This approach can also be useful for solving auxiliary optimization problems and for developing multimethod computational technologies.

Keywords: Global optimization, Multi-extremal function, Particle swarm algorithm, ChatGPT.

Predrag Stanimirovic[1], **Lev Kazakovtsev, Vladimir Krutikov**

Modified error functions in Continuous-time Recurrent Neural Networks

[1] University of Nis, Faculty of Sciences and Mathematics

Recurrent neural networks (RNN) form an important class of algorithms for solving various numerical linear algebra problems. Continuous-time RNN (CTRNN) use a system of ordinary differential equations to define inputs of incoming neurons. CTRNNs include two important classes: Gradient Neural Networks (GNN) and Zhang Neural Networks (ZNN). CTRNN uses a suitable error function $E(t)$ such that its zeros represent solutions of the considered optimization problem. The error function can be in the form of an array or a scalar. We study two approaches in improving CTRNN dynamic evolution. The first approach is based on applications of gradient-based methods for nonlinear minimization in defining $\dot{E}(t)$ and development of improved CTRNN design,

termed as GCTRNN. Particularly, the solution to the matrix equation $AXB = D$ is studied using the novel CTRNN models, termed as GCTRNN(A, B, D). The GCTRNN model is developed applying CTRNN dynamics on the gradient of $\dot{E}(t)$. The second approach to improve CTRNN dynamics is based on the hyperpower family of iterative methods with arbitrary order. Since the discretization of ZNN dynamics leads to the Newton iterative method, this study investigates a family of HoCTRNN dynamical models which are defined applying the CTRNN evolution on the sum of powers $E^i(t)$ defined following principles of hyperpower iterations of arbitrary order. The convergence analysis shows the asymptotical convergence of GCTRNN(A, B, D) and HoCTRNN(A, B, D) design to the least squares solution of $AXB = D$ that depends on the initial state matrix. The Simulink implementation of presented GCTRNN and HoCTRNN models is tested on the set of real matrices.

Keywords: Zeroing neural network, gradient neural network, gradient methods, hyperpower iterations.

Ilia Stepanov[1], **Daniil Musatov**[1]

Migrational Stability of Plane Tilings

[1] Moscow Institute of Physics and Technology

This paper studies a special case of the facility location problem applied to the standard Euclidean plane $\mathbb{R}^2$. The agents are uniformly distributed over the plane. The facility for each group is located at its geometric median in order to minimize the average distance between an agent and the facility. We study the equilibrium setting when no agent wishes to change their designated group in order to reduce their expenses (no one wants to selfishly migrate). We provide several useful tools and some results as to what a stable plane tessellation may look like. In particular, we describe all stable tessellations when all groups are triangles.

Keywords: Coalition formation, Plane tilings, Geometric median, Game theory, Facility location.

Alexander Strekalovsky[1]

On a Numerical Solution of Bilevel Optimization Problems

[1] Matrosov Institute for System Dynamics and Control Theory SB RAS

It is known that the bilevel optimization is now at the front edge of the recent advances in the modern mathematical optimization. Bilevel optimization problems are important theoretically and are very prospective in applications. Therefore, development of methods for solving various problems with hierarchical structure is one of the challenges faced by optimization theory and methods in the 21th

century. Moreover, problems with hierarchical structure arise in investigations of complex control systems and the bilevel optimization is the most popular modeling tool.

The bilevel optimization problem is not well posed if the inner (or lower level) problem does not have a unique optimal solution. This situation can be handled by using the optimistic or pessimistic (guaranteed) formulation of the problem. The work addresses some classes of bilevel optimization problems from a numerical point of view. First, we transform the original bilevel problem into a single-level nonconvex optimization problem. Then we apply the Exact Penalization Theory and Global Search Theory (GST) for d.c. optimization problems to the resulting problem.

The research was supported by RSF (project No 24-41-03004).

Keywords: Hierarchical problems, Bilevel optimization, Optimistic solution, Guaranteed solution, Exact Penalty approach, Global Search Theory, Numerical solution.

Alexander Strekalovsky[1]

ON THE GLOBAL SEARCH CONVERGENCE IN DC OPTIMIZATION WITH CONSTRAINTS

[1] Matrosov Institute for System Dynamics and Control Theory SB RAS

We address the DC minimization problem with DC equality and inequality constraints, which can be reduced (with the help of the Exact Penalty Theory) to another DC minimization problem without constraints [2]. Further, in [3] we developed a Global Search Scheme which consists of two stages: a) local search (LSM); b) numerical procedures, suggested by the Global Optimality Conditions (GOC), allowing to improve a current value $F_k(z^k) := F_{\sigma_k}(z^k)$ of the penalized cost function $F_\sigma(x) := f_0(x) + \sigma W(x)$, produced by special LSM, where $W(\cdot)$ is a penalty function and $\sigma > 0$ is a penalty parameter. In addition, the LSM produces a critical vector (with respect to LSM), which has to be approximate feasible in the original problem ($W(x^k) \leq \omega_k, \omega_k \downarrow 0$), and the corresponding value $\sigma_k > 0$. Under the assumptions of Theorem 4 [3], we were lucky to prove that

i) the sequence $\{z^k\}$ produced by the global search scheme (GSS) (with the resolving approximations, i.e. GSRS1) turns out to be minimizing to Problem $(P_{**}), (P_{**}) = (P_{\sigma **}), \sigma_{**} = \lim_{k\to\infty} \sigma_k, \{z^k\} \in \mathcal{M}(P_{**})$.

ii) When $\lim_{k\to\infty} W(z^k) = 0$, the sequence $\{z^k\}$ becomes minimizing to the original (with DC constraints) Problem (P).

It is worth noting that the penalty parameter σ_{**} turns out to be "exact", $\sigma_{**} \geq \sigma_{th}$ when $V(P_{**}) = V(P)$, so that (P) and (P_{**}) are equivalent in this sense. Hence, within and GSRS1 the LSM and the Global search procedures work

in coordination, as a teamwork, in the united computational process.

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Keywords: DC minimization, Exact Penalty Theory, Global Search, Local Search, Convergence.

Lijie Su[1], Lixin Tang, Peiyan Wang

Generalized Disjunctive Programming for Cutting Stock Problem in an Aviation Industry

[1] National Frontiers Science Center for Industrial Intelligence and Systems Optimization, Northeastern University; Key Laboratory of Data Analytics and Optimization for Smart Industry (Northeastern University), Ministry of Education, China

This paper addresses a two-dimensional cutting stock problem in an aviation industry, which characters include guillotine cutting, infinitely identical rectangular panels and multiple-size of items. The optimization objectives are maximization material utilization and efficiency of cutting operation. Using logic variables to represent assignment and relative positions of items, a convex Generalized Disjunctive Programming (GDP) model is formulated to describe this cutting stock problem. The operation constraints of guillotine cutting and superposition cutting are considered in the proposed GDP model. Aiming at the real large-scale instances, a Logic-based Outer Approximation method is designed to solve the GDP model, which integrates an initial heuristic and parallel computation of the subproblems. Numerical experiments are implemented based on real data from an aviation plant. The results show the validity of the proposed GDP model and the efficiency of Logic-based Outer Approximation method.

Keywords: Cutting Stock; Guillotine Cuts; Generalized Disjunctive Programming; Outer Approximation.

Andrei Torgashov, Oleg Snegirev

Comparative study of model predictive control algorithms based on linear and quadratic programming applied to industrial vacuum distillation unit

A comparative study of model predictive control (MPC) algorithms is considered in conditions when the parameters of the plant are uncertain. The algorithms of the MPC are compared according to which, at each control period, the problem of linear or quadratic programming is solved. The industrial vacuum distillation unit (VDU) for the production of vacuum gas oil and diesel fraction is investigated as controlled object. The stabilization of the main quality variables of the target products is simultaneously associated with difficulties in maintaining the required volume of the liquid phase in the side draw product zone of the VDU and may lead to a violation of the norms of process specifications. Therefore, the priorities of control goals and their consideration in the MPC-algorithms are also considered.

Keywords: model predictive control, linear programming, quadratic programming, petroleum refining, vacuum distillation unit, model uncertainty.

Anna Trubacheva[1]

About Some Models of Economic Growth

[1] Institute of Cytology and Genetics SB RAS

From 2021, income is more than 5 million rubles are taxed at a tax rate of 15%, with a rate of 13% for income less than the above amount [1]. The increased rate must be calculated from the total tax base. The choice at the government level of a progressive taxation scheme confirms the need to conduct research for economic growth models.

In [2] considers two basic taxation schemes – the uniform proportional tax, when the tax rate is constant, and a progressive tax, which depends on a size of the income, where the taxation rate is a step function. In the three-level hierarchical system "Government – Investor – Production" the tax scheme should allow the investor not to slow down the growth of production funds. It has been proven that in the case when there are no subsidies from the budget, the investor begins to reduce the optimal value of the capital endowment, which for a three-level system can lead to a decrease in gross product. It has also been proven that in the case of growth of production funds, the type of taxes does not affect the management structure. The report will present these results using models where all income is taxed.

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Keywords: growth economic models, management, production funds, flat scale,

progressive tax.

Nikolai Trusov[1], **Alexander Shananin**

Mathematical Modeling of the Consumer Loan Market in Russia

[1] Federal Research Center "Computer Science and Control" of RAS

The mathematical modeling of the economic behavior of the households is based on a Ramsey-type model. We introduce an optimal control problem that models the economic behavior of a representative household in an imperfect consumer loan market. The necessary optimality conditions are obtained in the form of Pontryagin's maximum principle in the Clarke form. A synthesis of optimal control can be constructed on an infinite time horizon. The synthesis includes special cases. We present a new model for the formation of interest rates on consumer loans. It is based on the analysis of the interests and logic of behavior of commercial banks that assess the risk of borrowers' default. According to the Feynman–Kac formula, the assessment is reduced to solving a boundary value problem for partial differential equations. It is possible to reduce the solution of the boundary value problem to the Cauchy problem for the heat equation with an external source and obtain a risk assessment in analytical form with a help of the Abel equation. The models of economic behavior of households in the consumer loan market and behavior of commercial banks are identified based on Russian statistics.

Keywords: mathematical modeling, optimal control synthesis, consumer loan, Feynman–Kac formula, Ramsey model.

Maksim Tsekot

Impact of different types of target functions on rational strategies for economic development

We consider two approaches to modelling the dynamics of income distribution for reproduction and consumption based on the Solow equation. One of the approaches is used in the recently proposed model [1]. The novelty lies in the fact that the subject's welfare level exceeds a fixed value during the whole period under consideration, and also in the fact that the control criterion is the minimisation of effort to achieve a given constraint on the welfare level. The second model under consideration is the simplified Ramsey model [2]. In it, the maximised functional is the "integral specific consumption". Some numerical calculations using the new approach were conducted and, unexpectedly, it was revealed that in the considered examples of medium-term forecasting, the value of "integral specific consumption" is higher than the value obtained with the simplified Ramsey model with the same values of parameters. The aim of this

report is to investigate in detail the influence of different types of target functions and different intervals of values of other parameters of mathematical models of growth of the economy on rational strategies of its development.

Keywords: two modelling approaches, new model, simplified Ramsey model, integral specific consumption, rational strategies for economic growth.

Oxana Tsidulko[1]

Uniform Capacitated Facility Location problem with soft capacities on tree-like graphs

[1] Sobolev Institute of Mathematics

In the network Uniform Capacitated Facility Location Problem with soft capacities (Soft-UCFLP) we are given an edge-weighted network graph $G = (V, E)$ with clients and facilities to open located at its vertices. Each client has an integer demand that must be served by one or several open facilities, each facility has an integer opening cost and an integer capacity. The term "uniform" in the problem name means that all capacities are identical, while "soft" means that one may open an arbitrary number of copies of each facility at its location, paying the opening cost for each copy. The weight of the shortest path between two vertices of G determines the cost of transporting a unit of product between them. The goal is to open a (multy-)set of facilities such that they jointly serve the full client demand, while the total transportation and facility opening cost is minimized.

In general graphs the problem is NP-hard and APX-hard even for the case of unbounded capacities. The most studied capacitated version of the problem in the literature is the classic CFLP with hard capacities in which at most one copy of each facility can be open. CFLP is NP-hard even on graphs with one vertex (if there can be several clients and facilities at one vertex), but can be solved in pseudopolynomial time on graphs of bounded treewidth. On the other hand, CFLP with uniform capacities admits a polynomial-time algorithm on paths [A. Ageev, 2009], but remains NP-hard on trees [A. Ageev et al., 2022]. Regarding constant factor approximation algorithms, CFLP and UCFLP were intensively studied in the case of general input graph and unit client demands. The current best approximation factors are 5 for CFLP [M. Bansal et al., 2012] and 3 for UCFLP [A. Aggarwal et al., 2013].

Soft-CFLP has also been studied in the literature, mainly from the approximation point of view, from which it seems to be easier than CFLP: Soft-CFLP with unit client demands in general graphs admits a 2-approximation algorithm [M. Mahdian et al., 2006]. Yet, to the best of our knowledge, the complexity of Soft-CFLPs on less general graph classes has not been previously analysed. It is easy to see that Soft-CFLP with arbitrary capacities is also NP-hard even on graphs with one vertex, and the pseudopolynomial-time algorithm for CFLP on graphs

of bounded treewidth can be easily adapted to solve Soft-CFLP. For the case of uniform capacities, the results are not as straightforward. In this work, we consider the problem on tree-like graphs and show that Soft-UCFLP is not much easier than UCFLP in this sense: it can be solved in polynomial time on paths, and is NP-hard on trees.

Keywords: facility location, soft capacities, polynomial-time algorithm, NP-hardness, trees, paths.

Alexander Turnaev[1], **Artem Panin**[1]

Stochastic Greedy Algorithms for a Temporal Bin Packing Problem with Placement Groups

[1] Sobolev Institute of Mathematics

The paper considers a temporal bin packing problem, where bins are servers using the Non-Uniform Memory Access architecture, and items are virtual machines. Bins are grouped together into racks. The main difference from the classical bin packing problem is that items are organized into placement groups, which are subdivided into subgroups. Items from different subgroups of the same group conflict and cannot be placed on the same rack.

The additional constraints considered are relevant to cloud computing. Service reliability is essential for both service providers and their customers. This is achieved by isolating subgroups of virtual machines from the same placement group within a failure domain.

Several stochastic algorithms have been developed to solve this problem. They are based on the classical first-fit algorithm, reordering of the packing sequence, and the bisection method. The algorithms give good results even for a rather naive initial solution (ordering). Moreover, they are easily parallelizable, which allows them to have an acceptable speed even for large problems.

Keywords: stochastic algorithms, temporal bin packing, first-fit, placement groups, placement constraints.

Inna Urazova, Ruslan Simanchev[1]

Valid inequalities for single machine scheduling problem with preemptions

[1] Omsk State Univercity

The problem of minimizing the total time for servicing various jobs by one machine with preemptions is considered. The processing time is discrete and same for each jobs. The complexity status of this problem is unknown. We describe a set of service schedules in terms of the polyhedral approach. We present new valid and support inequalities for the polyhedron of this problem. Using new

classes of inequalities, several polyhedral relaxations are obtained. The results of a computational experiment based on the analysis of these relaxations are presented.

Keywords: scheduling, polyhedral approach, experiment.

Evgenia Ushakova[1]

HOTELLING'S DUOPOLY WITH INFORMATIVE ADVERTISING ON THE PLANE

[1] Dostoevsky Omsk State University

The model of Hotelling's duopoly on the plane with informative advertising is considered. The customers are uniformly distributed. Consumers are uniformly distributed insaite the circle. Transportation costs are given by the Manhattan metric or the quadratic function. The function of advertising costs is investigated in two variants: quadratic and logarithmic. The equilibrium price is determined for each model. The equilibrium level for advertising is also calculated in the situation of symmetrically placed sellers. A numerical solution is presented for all cases with different threshold values of advertizing and different variants of sellers' locations.

Keywords: Hotelling's duopoly on the plane, informative advertising, equilibrium prices.

Andrey Uskov, Valentina Kotezhekova

A REAL-WORLD PARCEL ROUTING PROBLEM: MIP FORMULATION AND HEURISTIC

In the paper the optimization problem of parcel delivery is considered. This belongs to the family of Multi-Commodity Network Flow Problems (MCNF). The problem is characterized by a significant number of incorporated business requirements (different vehicles types and transport routes, due dates, multiple transit stations, etc.) together with a wide data scope covering thousands of post offices across a country and millions of daily parcels which both lead to a large problem size. The problem is formulated in terms of a Mixed Integer Linear Program with over 3 billion variables and over 800 thousand constraints. A heuristic-based approach in conjunction with the use of a state-of-the-art MIP solver is proposed to solve real-life problem. Computational results demonstrates that the designed heuristic search is able to provide good solutions for large-scale instances of the MCNF in a reasonable time.

Keywords: Multi-Commodity Flow Network Optimization, Large-Scale Routing, MILP, Heuristics, Brunch-And-Bound.

Vyacheslav Ustyugov[1]

On different methods for automated MILP solver configuration

[1] Sobolev Institute of Mathematics

Mixed Integer Linear Programming (MILP) solvers, such as GUROBI, besides problem input may also receive a set of tunable parameters that affect the performance of a solver. Although iterated racing methods (namely Irace) show significant success with such tasks, we still consider local-search methods competitive. Therefore, we investigate different sides of methods' capabilities, such as real-time efficiency, number of solver runs, etc. Further research will define necessity for a wide-scale automatization, namely usage of machine learning and neural network applications for feature extraction based on task files and prediction of parameter configuration based on obtained features using general linear model. The research was conducted for a task-scheduling problem.

The research is supported by Russian Science Foundation, grant N22-71-10015, https://rscf.ru/en/project/22-71-10015/.

Keywords: MILP, GUROBI, VNS, local search, Neural networks.

Stepan Utyupin, Andrey Melnikov[1]**, Vladimir Beresnev**

Stability of vertex covers in a game with a finite number of steps

[1] Sobolev Institute of Mathematics

The eternal vertex cover problem is a variant of the graph vertex cover problem that can be represented as a dynamic game between two players (Attacker and Defender) with an infinite number of steps. At each step, there is an arrangement of guards over the vertices of the graph, forming a vertex cover. Attacker attacks one of the graph's edges, and Defender must move the guard along the attacked edge from one vertex to another. In addition, Defender can move any number of other guards from their current vertices to some adjacent ones to obtain a new vertex cover. Attacker wins if Defender cannot build a new vertex cover after an attack

In this paper, we propose a procedure that allows us to answer the question whether there is a winning strategy for Defender that allows protecting a given vertex cover within a given finite number of steps. To construct a strategy of Defender, the problem is represented as a dynamic Stackelberg game, at each step of which, the interaction of the opposing sides is formalized as a two-level mathematical programming problem. The idea of the procedure is to recursively check the 1-stability of vertex covers obtained as a result of solving the lower-level problems, and to use some information about the covers already considered.

Computational experiments demonstrates results from which it can be assumed that most random covers either have a high degree of stability or are

unstable.

Keywords: eternal vertex cover, graph algorithms, graph protection, bilevel programming, dynamic Stackelberg game.

Stepan Vasilev[1], **Oleg Khamisov, Petr Vasilev**

Short-Term Voltage Instability Identification: A Combined Approach of Maximum Lyapunov Exponent and K-Means Clustering

[1] Skolkovo Institute of Science and Technology

This study is focused on the combination of Maximum Lyapunov Exponent (MLE) and k-means clustering for the short-term voltage instability (STVI) identification. MLE and clustering are applied to a class of differential systems that describe electromechanical and electromagnetic processes in multi-machine power grids. The authors contribute by employing this combined method for automatic labeling of voltage measurement sets, allowing the differentiation of transient regimes in the power system (PS), which can distinguish STVI among switching events in normal operational mode, faults. The obtained results can be utilized for comprehensive monitoring of transient regimes in the PS. We support our results with numerical experiments in PSCAD simulation software.

Keywords: short-term voltage instability, maximum Lyapunov exponent, clustering, PSCAD.

Igor Vasilyev[1], **Tatiana Gruzdeva**[1], **Maria Barkova**[2], **Denis Boyarkin, Dmitrii Iakubovskii**

On Active-Set Methods for Quadratic Problems with Positive Semidefinite Matrices

[1] Matrosov Institute for System Dynamics and Control Theory SB RAS

[2] Matrosov Institute for System Dynamics and Control Theory of Siberian Branch of Russian Academy of Sciences

Quadratic programming problems (QP) arise in a vast variety of real-world applications in finance, engineering, operational research, and many other fields. A huge number of methods and their various modifications have been developed for solving convex QPs. One of the first algorithms popularized as the solution method for QPs was the active-set method. The main idea of the method is selection of a set of binding constraints (an active set) and then iteratively adaptation it by adding and dropping constraints from the index set. The dual active-set (DAS) method of Goldfarb and Idnani is still well-known to be efficient and numerically stable. Taking advantage of the fact that the unconstrained minimum of the objective function can be used as a starting point of the method, its implementation utilizes the Cholesky and QR factorizations and procedures

for updating them. At the same time, it suffers from the restriction that it cannot be applied to problems with positive semidefinite objective matrices, whereas in practice optimization problems may often be of such structure. We propose an add-in to the method of Goldfarb and Idnani that allows solving problems with positive semidefinite matrices directly by the DAS algorithm with a warm-start where a good estimate of the optimal active set is used to start the algorithm. The proposed technique is based on the well-known possibility of representing any matrix, as a difference of two, positive definite ones and solving then a sequence of so-called partially linearized problems. It can be viewed as a particular case of the special local search method for DC minimization problems. A computational experiment to testing the proposed approach was carried out on QPs with positive semidefinite objective matrices from the Maros and Meszaros test set.

The research of T. Gruzdeva and M. Barkova was supported by RSF (project No 24-41-03004).

Keywords: Quadratic programming, Positive semidefinite matrices, DC function, Local search, Dual active-set method, Warm-start.

Igor Vasilyev[1], Anton Ushakov[2], Dmitry Arkhipov, Ivan Davydov[3], Ildar Muftahov, Maria Lavrentyeva

FAST HEURISTICS FOR A STAFF SCHEDULING PROBLEM WITH TIME INTERVAL DEMAND COVERAGE

[1] Matrosov Institute for System Dynamics and Control Theory SB RAS

[2] Matrosov Institute for System Dynamics and Control Theory of SB RAS

[3] Sobolev Institute of Mathematics

Staff scheduling is a key component of supporting and increasing competitiveness for many service enterprises. This is of especially urgent concern for organizations that provide service on a twenty-four hour basis and often encounter significant fluctuations of demand. By scheduling personnel, the employers have also to strictly follow local laws, industrial regulations, and workload agreements that may considerably affect the final schedule. Staff preferences have also to be taken into account when planning work schedules, since it may reduce turnover and increase productivity. In this paper we consider a staff scheduling problem that arise in the industrial fields where the demand in staff is highly dynamic and varies within time intervals throughout a day. The goal is to assign each employee with a shift for each day of a planning horizon so as to minimize the sum of unsatisfied demand over all time intervals subject to hard workplace constraints. Note that each employee may have his/her day-specific set of pre-defined shifts and a set of work-rule constraints. We formulate the scheduling problem as a mixed-integer program. We develop several fast two-stage heuristic algorithms

that includes a constructive step to find initial solution followed by fast local search procedures. We demonstrate the effectiveness of the proposed approaches on a number of real-world huge-scale scheduling problems involving thousands of employees.

Keywords: staff scheduling, rostering, local search.

Alexander Vasin, Olesya Grigoreva[1]

On the optimal management of energy storage

[1] Lomonosov Moscow State University

In the present work, we consider a problem of optimizing storage management for a storage device, whose actions do not affect the market prices of electricity. An owner of the device can buy energy at fixed tariffs or at wholesale market prices and sell it back to the network at these prices. He aims to maximize his profit from the energy resale in a given planning interval. We obtain an efficient algorithm for calculating the optimal storage strategy. This algorithm permits to search for a solution to the problem in an "almost analytical" form. The solution is of interest for consumers using storage devices, as well as for a centralized regulation of electricity networks.

Keywords: Energy storages, Optimal control, Efficient algorithms.

Maksim Vodyan

Metaheuristics for finding the stability radius in the bilevel facility location and mill pricing problem

This paper studies the threshold stability of the location and pricing problem. The initial problem is formulated as a leader-customer Stackelberg game, in which the leader locates facilities producing homogeneous products and sets prices for each of the open facilities using the mill pricing strategy. Each customer has a budget and unit demand. They choose the facility that achieves the minimum of their total costs (transportation and product purchase costs) and make a purchase if their costs do not exceed their budget. The goal is to maximize the leader's revenue. In the threshold stability problem, it is necessary to choose a feasible solution to the location and pricing problem and such a maximum deviation from the current budgets of customers that the leader's revenue will not be less than a predetermined value (threshold). The threshold stability of location problems with uniform and discriminative pricing has been previously studied. The new formulation with the mill pricing turns out to be computationally more difficult since the pricing subproblem is NP-hard when the location of facilities is fixed. This raises the question of whether efficient approximate algorithms for this more hard threshold stability problem can be developed using a previously developed

approach that has proven to be very effective for simpler formulations. In this work, several approximate algorithms for solving the new threshold stability problem are developed. The idea of these algorithms is to find an approximate value of the radius of threshold stability based on the optimal solution of the initial problem with a fixed location of facilites. The algorithms differ in the way different locations of facilites are compared, which ultimately leads to different estimates of the radius of threshold stability. The numerical experiment showed the effectiveness of the chosen approach to constructing approximate solutions for the investigated problem, both in terms of the algorithms' runtime and the quality of the obtained solutions. The work was carried out within the framework of the state contract of the Sobolev Institute of Mathematics (project FWNF2022-0019).

Keywords: bilevel programming, facility location, mill pricing, threshold stability problem, stability radius, vnd heuristic.

Xin Wang, Liqun Wang, Lingchen Kong

ESTIMATION OF SPARSE COVARIANCE MATRIX VIA NON-CONVEX REGULARIZATION

Estimation of high-dimensional sparse covariance matrix is one of the fundamental and important problems in multivariate analysis and has a wide range of applications in many fields. This paper presents a novel method for sparse covariance matrix estimation via solving a non-convex regularization optimization problem. We establish the asymptotic properties of the proposed estimator and develop a multi-stage convex relaxation method to find an effective estimator. The multi-stage convex relaxation method guarantees any accumulation point of the sequence generated is a first-order stationary point of the non-convex optimization. Moreover, the error bounds of the first two stage estimators of the multi-stage convex relaxation method are derived under some regularity conditions. The numerical results show that our estimator outperforms the state-of-the-art estimators and has a high degree of sparsity on the premise of its effectiveness.

Keywords: Multi-stage convex relaxation method, Non-convex regularization, Sparse covariance matrix.

Lei Wang, Hongwei Gao[1]

IRRATIONAL-BEHAVIOR-PROOF CONDITIONS FOR STOCHASTIC GAMES OVER EVENT TREES

[1] Qingdao University

In this paper, the irrational-behavior-proof conditions in a class of stochastic dynamic games over event trees are presented. Four kinds of irrational-behavior-

proof conditions are proposed by the imputation distribution procedure, and their relationships are discussed. More specific properties for the general transformation of characteristic functions are developed, based on which, the irrational-behavior-proof conditions are proved to be true in a transformed cooperative game.

Keywords: Dynamic cooperative game; Event tree; Irrational-behavior-proof condition; Imputation distribution procedure; Transformation of characteristic function.

Zaiwen Wen[1]

Exploring the Learning-Based Optimization Algorithms

[1] Peking University, China

The recent revolutionary progress of artificial intelligence has brought significant challenges and opportunities to mathematical optimization. In this talk, we briefly discuss two examples on the integration of data, models, and algorithms for the development of optimization algorithms: ODE-based learning to optimize and learning-based optimization paradigms for solving integer programming. We will also report a few interesting perspectives on formalization and automated theorem proving, highlighting their potential impact and relevance in contemporary mathematical optimization.

Keywords: artificial intelligence, integer programming, automated theorem proving.

Jian Wu, Lixin Tang, Shiyan Huang, Xingyu Wang

Convex Cluster for functional data

In typical industrial hot rolling production, complex data such as functional data and chemical composition are often encountered. This poses significant challenges for industrial data analysis, including clustering and regression. Based on knowledge of metallurgy and the mechanism of steel rolling, a new two-stage convex clustering method with complex data is proposed in this paper. In the first stage, using spline coefficients to describe the characteristics of the function and performing IRL transformation on compositional data. In the second stage, consider the mechanism knowledge of rolling process, a new distance is proposed. The function data and compositional data are projected to the euclidean space, Using the convex relaxation method, the K-means clustering is remodeled as a convex optimization problem, and the alternating minimization algorithm is used to solve it. In addition, the theoretical properties of the clustering algorithm are established, and the comparison with other methods and the case study also confirmed the effectiveness of the method

Keywords: Functional Data, Convex Cluster, Hot Rolling.

Zhouming Wu, Yifen Mu, Xiaoguang Yang, Ge Chen

Solve the NE via dynamics of fictitious play for a specific stochastic game

Stochastic games are widely used to model complex sequential interactions with the random change of the environment. General NE in stochastic games is very hard to solve. Previous work(Schoenmakers et al 2007) showed that for non-zero-sum stochastic games the famous fictitious play process does not necessarily converge for the simplest model with only 2 players, 2 states, 2 actions for each player and a unique stationary equilibrium by proving the behavior of FP dynamics. In this paper we try to reveal the hidden information of the FP dynamics for the stochastic game and give a feasible method to find NE in the term of stationary strategies. This method has been proven effective in normal form games in our previous work. The investigation of this paper implies the great potential of the NE solving method based on the detailed analysis of learning dynamics for repeated games.

Keywords: NE solving, stochastic games, fictitious play, dynamical systems.

Xiangru Xing, Xin Wang, Lingchen Kong

Multi-view clustering optimization with group sparse

Multi-view clustering leverages consistent and complementary information across multiple views to provide more comprehensive insights than analysis of single-view data. In practical applications, the heterogeneity and redundancy of high-dimensional multi-view data pose many challenges to the existing clustering methods. In this paper, we propose a new multi-view clustering method with adaptive group sparsity for high dimensional mixed data types, which can extract effective features and induce intra-group sparsity. For the mixed multi-view dataset with different distribution of features, we use different loss or divergence and adopt a joint fusion penalty to obtain common groups. Different from the traditional convex regularization terms, the informative features and samples are screened by utilizing group sparse with l_0 norm to improves the robustness of the model. In addition, we develop the equivalence of local minimum points and KKT points for our model, and employ a ADMM algorithm to solve it. A large number of numerical experiments on simulated and real data show that the proposed method has excellent performance in clustering and feature selection.

Keywords: Multi-view clustering, Mixed multi-view data, Non-convex regularization, Group sparse, Feature selection..

Demyan Yarmoshik[1], **Michael Persiianov**

On the Application of Saddle-Point Methods for Combined Equilibrium Transportation Models

[1] MIPT

Travel demand modeling is an essential tool in urban planning and transportation system management. Existing practically efficient algorithms for solving multistage travel demand problems are variations of the heuristic sequential procedure. We propose a novel approach that applies saddle-point methods to a combined convex optimization formulation of the problem. Unlike all previous methods, our algorithm does not require costly shortest-paths calculations, and can be seamlessly scaled on GPUs. We show that in some cases it drastically outperforms the sequential procedures.

Keywords: Convex optimization, Trip distribution, Traffic assignment, Combined transportation model, First-order methods, Affine constraints.

Rashid Yarullin[1], **Igor Zabotin, Oksana Shulgina**

A Relaxed Cutting Method for the Convex Programming Problem

[1] Kazan (Volga region) Federal University

We propose a method for constrained minimization of a convex non-differentiable function, which belongs to the class of cutting methods. To construct iteration points, it uses the immersion operation into polyhedral sets of both the constraint set of the original problem and the epigraph of its objective function. The method is characterized by the fact that the main sequence of iteration points is constructed belonging to an admissible set with a relaxation condition. In this regard, it is permissible to check each iteration point for e-optimality. In addition, the method allows for combination with other relaxation algorithms. The convergence of the proposed method is substantiated and its implementations are described.

Keywords: convex programming, iteration point, approximating set, cutting plane, epigraph of a function, convergence, e-solution, constrained minimization.

Marina Yashina[1], **Alexander Tatashev, Ivan Kuteynikov**[1]

Qualitative behavior of the multi-scaled Buslaev chainmail

[1] Moscow Automobile and Road Construction State Technical University (MADI)

Prof. A.P. Buslaev's works established the fundamental methods for describing flows on traffic networks. In accordance with the approach the model is considered as a discrete dynamical system, in which the motion of particles on a carrier, which is a system of circuits with a network structure, is carried out. The constraints imposed on the network structure and movement rules allows the

systems to be studied analytically. In previous works, a one-particle two-dimensional dynamical network, called a chainmail, with four cells was considered. It was proved that for a toroidal closed and rectangular open chainmail with co-directional and one-directional particle movement, the state of the system depends on the topology and the movement direction. In the presented work, a generalized version of the multi-scaled chainmail with an arbitrary number of intermediate cells and a cluster of particles on the circuits is investigated. Results are obtained and hypotheses are formulated for different variants of the chainmail.

Keywords: Dynamical systems, Self-organization, Spectrum, Circuit model, Multi-scaled chainmail.

Xiaohui Yu, Qiang Zhang, Mengying Xiong

THE EQUAL SURPLUS DIVISION BASED ON THE BARGAINING POWER OF PRIOR COALITION IN THE COALITION STRUCTURE GAME

As a member of prior coalition, the motivation for a player to participate in the grand coalition is to get more payoff value, so the scale of the prior coalition needs to be considered in the allocation among the prior coalitions. Considering the size difference of prior coalitions, two extensions of ESD value are given, which are the weighted equal surplus division value and the equal surplus division value.

Keywords: surplus division, coalition, game.

Wenwu Yu[1]

"DISTRIBUTED OPTIMIZATION +" IN NETWORKS: A NEW FRAMEWORK

[1] Southeast University, China

Distributed optimization is solved by the mutual collaboration among a group of agents, which arises in various domains such as machine learning, resource allocation, location in sensor networks and so on. In this talk, we introduce two kinds of distributed optimization problems: (i) the agents share a common decision variable and local constraints; (ii) the agents have their individual decision variables but that are coupled by global constraints. This talk comprehensively introduces the origin of distributed optimization, classical works as well as recent advances. In addition, based on reinforcement learning, shortest path planning, and mixed integer programming, we build the distributed optimization framework of distributed optimization, and also discuss their applications. Finally, we make a summary with future works for distributed optimization.

Keywords: distributed optimization, group of agents, reinforcement learning.

Alexander Yuskov[1]**, Igor Kulachenko, Andrey Melnikov**[2]**, Yury Kochetov**[2]

Stadium antennas deployment optimization

[1] Novosibirsk State University

[2] Sobolev Institute of Mathematics

The stadium is divided into sectors. Each sector is split into cells. Users in the cells must be provided with a certain quality of signal from antennas assigned to their sector. Our goal is to select antenna types, their location, assignment to sectors, and orientation to optimize the signal distribution, measured by three different metrics under some technical constraints. The quality metrics are signal quality, average signal-to-interference ratio (SIR), and consistency. Each variant of antenna deployment is evaluated by a simulator. Thus, we deal with a constrained black-box optimization problem with three objectives. To tackle the problem, we design a three-stage algorithmic approach. In the first stage, we apply a fast constructive heuristic. Later on, a local improvement procedure is called. Finally, a VNS metaheuristic is used to get high-quality solutions. The approach demonstrates strong performance and ability to improve signal quality by 7% and SINR by at least 14% without worsening the given consistency threshold for test instances with up to 7 antenna types, 19 sectors, and 4426 cells.

Keywords: Black box optimization, Simulation, VNS metaheuristic, Wireless network.

Gennady Zabudsky[1]

Maximin and Maxisum Network Location Problems with Various Metrics and Minimum Distance Constraints

[1] Sobolev Institute of Mathematics SB RAS

Some facility location problems on a road network connecting several settlements are considered. The facility has adverse effects to people of the settlements. The effects are decreased with increasing a distance from the facility to a settlement. The problems with criteria for maximization the minimum distance to nearest settlement (maximin) and maximization a sum of the distances from the facility to the settlements (maxisum) are investigated. The constraints to the minimum admissible distances from the settlements to the facility and a budget of transportation costs for servicing the settlements by the facility are given. Euclidean metric is used in the objective functions and in the minimum admissible distances. The shortest paths metric is used in calculating of the transportation costs. Polynomial algorithms for finding all local optimums of the problems are proposed.

Keywords: network location, polynomial algorithm, minimum distance constraints.

Aleksey Zakharov[1], **Yulia Zakharova**[1]

Integer Programming Models and Metaheuristics for Customer Order Scheduling

[1] Sobolev Institute of Mathematics SB RAS

The problem of scheduling customer orders in a production unit is investigated. The order of a customer consists of several products. When the unit is switched from one product to another a setup operation arises. We consider two criteria: the total completion time and the weighted number of orders completed by their due dates. We provide and compare several approaches to construct integer linear programming models of the problem. A memetic algorithm with optimized operators is proposed for searching near optimal solutions. The results of the experimental evaluation are analysed on a series of instances and compared with state-of-the-art metaheuristics.

The research was supported by Russian Science Foundation grant N 22-71-10015.

Keywords: scheduling, algorithm, model, experiment.

Yulia Zakharova[1], **Lidia Zaozerskaya**[1]

Large neighborhood search for split delivery vehicle routing problem with time windows

[1] Sobolev Institute of Mathematics SB RAS

A vehicle routing problem for servicing objects with the possibility of distributing work between vehicle taking into account time windows is considered. Large neighborhood search algorithms with different types of neighborhoods are developed. Here an improving solution is constructed using the "destroy" and "repair" methods at each step. The "repair" operators are based on solving mixed integer subproblems of reduced dimentions. An experimental evaluation shows that the algorithms demonstrate competitive results.

Keywords: vehicle routing, local search, integer programming, experiment.

Li Zhang

Mathematical Model of Product formula for Non-woven Fabric Production Engineering and The Corresponding Optimization Computation

Non-woven fabric production is common in Suzhou area, but the traditional formula is not fine when the enterprise technical department using the experience of sample production Time is long, cost is high, and pollution is serious. For this problem, combined with the specific data and production process of the company in Suzhou area, this paper first considers the brown product data with multiple linear regression analysis, it is found that the test effect is not

good. So it is necessory to use other methods to solve this problem. Considering the characteristics of non-woven production the problems, after careful analysis and discussion, it is key to set up a suitable grid envelope mathematical model for non-woven production problem and design the core algorithm of non-woven production formula ratio calculation. Through the idea of numerical optimization, we establish the equation group and calculate the change speed of each element of formula composition. Considering the formula composition, we establish the optimization model of formula composition calculation, set the range of parameter changes and put forward the corresponding algorithm for calculation. Through the specific data calculation and analysis, it is found that the model is reasonable and efficient and the enterprise application is good.

Keywords: Non-woven fabric production; Production formula; Optimization algorithm; Big data analysis.

Chi Zhao[1], **Elena Parilina**[1]

NETWORK STRUCTURE PROPERTIES AND OPINION DYNAMICS IN TWO-LAYER NETWORKS WITH HYPOCRISY

[1] Saint Petersburg State University

We extended a social Zachary's karate club network by adding the second layer or internal layer of communication. Different models of opinion dynamics are represented on such two-layer network. The presence of internal layer is motivated by the fact that individuals can share their real opinions with their close friends. In external layer, individuals express their opinions publicly. Starting from a real structure of the external layer defined as Zachary's karate club, we observe how opinion dynamics is affected by internal layer, i.e. how consensus time and winning rate change depending on an internal layer structure. We find significantly strong correlation between internal graph density and consensus time, as well as group degree centrality and consensus time.

Keywords: Opinion Dynamics, Voter Model, Concealed Voter Model, General Concealed Voter Model, Zachary's karate club.

Jiangjing Zhou[1], **Vladimir Mazalov**[2]

DYNAMIC STABILITY OF COALITION STRUCTURES IN NETWORK-BASED POLLUTION CONTROL GAMES

[1] Karelian Research Center of Russian Academy of Sciences

[2] Karelia research center

This paper investigates the dynamics of coalition stability in pollution control games that are built on networks. It specifically focuses on the concept of dynamically stable coalition partitions involving three asymmetric players. At first, the

research presents the model and clearly identifies optimal strategies for players in various coalition forms. In order to evaluate the dynamic stability of these coalitions, the system additionally calculates the time-consistent Imputation Distribution Procedure (IDP) for players belonging to different coalitions. This work presents a novel idea of dynamically Nash stable coalition partitions, which is defined using the IDP, in contrast to the traditional definition. By accurately identifying the IDP as specified, we may attain dynamically Nash stable coalition partitions in the simulation results.

Keywords: Dynamically Nash stable coalition partitions, Network-based pollution control games, Asymmetric players.

Tatiana Zvonareva[1], **Olga Krivorotko**

MEAN FIELD CONTROL PROBLEM FOR MATHEMATICAL MODEL DESCRIBING INFORMATION DIFFUSION IN ONLINE SOCIAL NETWORKS

[1] Sobolev Institute of Mathematics SB RAS

The process of information diffusion in online social networks characterizes by nonlinear diffusion logistic equation [1]. Adding control parameter that provides a Nash equilibrium in the system of interacting agents and minimizes the cost functional transforms to the mean-field games. We consider a large number of users who can take states from 0 (means that the user is involved in the process of information dissemination) to 1 (means the opposite). Then the density of users obeys the Kolmogorov-Fokker-Planck equation. Using the Lagrange multiplier method [2] a system similar to the Hamilton-Jacobi-Bellman equation and the optimality conditions are constructed. Such problem was solved by the finite-difference scheme proposed in [3]. The numerical calculations were analyzed and discussed. To solve the inverse problem, we need to determine the initial density and control functions from the additional measurements in computational domain. The sensitivity-based identifiability analysis was conducted. The work is supported by the Mathematical Center in Akademgorodok under the agreement No. 075-15-2022-281 with the Ministry of Science and Higher Education of the Russian Federation.

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Keywords: Kolmogorov-Fokker-Planck equation, Hamilton-Jacobi-Bellman equation, social networks, source problem, mean field control, identifiability.